OPTIMIZING TOLERANCE ALLOCATION FOR NORMALLY DISTRIBUTED DIMENSIONS INCLUDING PROCESS CAPABILITY CONSTRAINTS

A Dissertation

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Doctor of Philosophy

Industrial Engineering

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by

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To the Graduate School:

This dissertation entitled "Optimizing Tolerance Allocation for Normally Distributed Dimensions Including Process Capability Constraints" and written by Piangjai Panichkun is presented to the Graduate School of Clemson University. I recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy with a major in Industrial Engineering.

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ABSTRACT

Tolerance allocation, which is a phase of product design, can reduce cost of the product without any investment or improvement in the quality of the product or the process while the product still satisfies all the specified requirements. Optimizing dimensional tolerance allocation was examined for areas of potential improvement. Current research in the field of optimizing tolerance allocation has proposed various models for specific conditions with limitations in application. This research develops a model for optimizing tolerance allocation that can be solved using common off-the-shelf software, and applying design of experiments to analyze the sensitivity of the total cost to cost coefficients and constraints.

The model proposed can be applied to optimizing dimensional tolerance allocation for normal distributions with any combination of three characteristics. They are (1) the process mean either equal or unequal to the nominal size, (2) possible unbalance in the upper and the lower quality loss coefficients, minimum requirements for the process capability indices, specified ranges for semi-tolerance zones, and/or specified minimum proportions of conforming units of the product, and (3) ability to select among non-inspection (NI), 100% inspection without reworking (IWR) or 100% inspection with imperfect reworking strategy (IIR).

The proposed model assumes that the process standard deviation for each part has an increasing linear relationship with its tolerance. The minimum total cost is the criterion for the optimization, solved by genetic algorithm in Evolver. The proposed

model is subject to constraints associated with (1) the process standard deviation of the gap resulting from the square root of the summation of the parts and the envelope variances, (2) the allowable ranges for the semi-tolerance zones for the parts, (3) the specified minimum requirements for process capability indices, (4) the allowable ranges for process standard deviations of the parts, and (5) the specified minimum proportion of conforming units of the product.

Design of experiments (DOE) is applied to sensitivity analysis to completely finish the process of optimization. This approach can significantly reduce the number of experimental runs while it can analyze the sensitivity of the factors with a specified level of significance.

DEDICATION

I dedicate this dissertation to my family, especially to my mother. She understood, encouraged and supported me with her love and patience for the five and a half years while I was working on my Ph. D. I finished this research because of a promise to my father who passed away on November 15, 1997. In addition, my sisters, brothers, nieces and nephews greatly supported and encouraged me to finish this research.

I was lucky to be adopted by Mrs. Pitsa R. Johnson during my study at Clemson University. She is a generous woman, who patiently helped me to improve my English. I would like to dedicate this dissertation to her as well.

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I am also sincerely thankful to my other committee members, Dr. Brian J. Melloy, Dr. Byung-Rae Cho and Dr. William G. Ferrell. I would like to express my appreciation to Dr. Peter R. Nelson. My research would not have been completed without his patient and useful advice. I would like to express my sincere gratitude to the Director of the Writing Center, Mrs. Barbara J. Ramirez, who spent a lot time helping me write this dissertation despite her very busy schedule. Finally, I would like to thank the Royal Thai Government for its financial support. I would not have had the opportunity to study in the United States of America without the scholarship sponsored by my beloved nation, Thailand.

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CHAPTER 1

INTRODUCTION

Problem Context

A product is considered to be composed of components assembled in an envelope. The dimension for each part and that for the envelope slightly vary for each of the units produced due to the random causes of variations, even though the assignable causes of the variations have been removed. Therefore, not only the nominal sizes but also the tolerances need to be specified. Normally, the nominal size and the tolerance for a product are specified first, and then those for the parts are determined subject to the given conditions. Tolerance allocation (i.e. determining the optimum tolerances for the parts based on the specified tolerance for the product) but not tolerance analysis (i.e. determining the optimum tolerance for the product based on the specified tolerances for the parts) is applied to most products. The interest of this research is optimizing (linear dimensional) tolerance allocation; that is, determining the optimum tolerances of the parts, whose dimensions are linearly assembled in the envelope with the surfaces of all dimensions parallel to one another, based on the specified standard deviation of the envelope and the allowable maximum standard deviation of the gap of the product.

Mostly, optimizing (linear dimensional) tolerance allocation deals with two-sided tolerances, the lower and the upper semi-tolerance zones. Semi-tolerance zones are the amounts by which dimensions may vary between the nominal size and the respective upper or lower specification limit. In most cases this dimensional variation has a normal distribution. In addition, some parts have (1) the process means of the dimensions offset from the nominal sizes, (2) quality loss coefficients for the upper sides different from those for the lower sides, and/or (3) customers' requirements and/or conditions for proper functioning making the lower semi-tolerance zones different from the upper semi-tolerance zones. Furthermore, the production costs for major manufacturing processes have been studied as functions of tolerances. Thus, appropriate allocation of tolerances is an important issue in the economics of manufacturing.

Problem Importance

The current research in the field of optimizing tolerance allocation uses simplistic models with many approximations and assumptions. For instance, (1) they independently approximate the skewed distribution of the dimension for each side of each part with a singly truncated normal distribution or with a triangular distribution, (2) they assume that the process means in production could be adjusted to equal the values determined from their models even though those are economically impractical, (3) they do not independently consider semi-tolerance zones and/or quality requirements for the upper and the lower sides for the objective functions and/or the constraints even though the optimum values for the lower semi-tolerance zones are different from those for the upper, and (4) they use simplistic production cost-tolerance models for the objective functions. The solutions determined from the simplistic models with many approximations and assumptions may be impractical for real application or may be economically impractical, and can be far from the true optimum values, with their total costs higher than those could be. However, the models having very highly descriptive objective functions, e.g.

the production cost-tolerance models with fifth-order polynomials and the fourth-order B-Spline curves, are difficult to solve.

The current models do not integrate the minimum process capability indices into their constraints, although these indices are sometimes specified by customers. They are required because they compare the parameters of the process with the specification limits, with the specification limit and the nominal size, or with the specification limit and the process mean. This allows the use of only one value (the index) for each process in comparing the capabilities of various processes despite different product types. Another important deficiency of the current models is that the models do not convey sufficient information for some terms. For example, the current models do not clearly define what terms should or should not include the effects of (1) means offset from the nominal sizes, (2) inspection strategies chosen, and (3) dimension truncations. Therefore, the errors due to misapplications of some terms for the current models result in non-optimum solutions making those have total costs higher than what they could be. The model being proposed in this research can overcome all of the mentioned disadvantages (except optimizing tolerances for the dimensions with skewed distributions) of the current models; therefore, it can achieve design goals at lower cost.

Research Goals

The model for optimizing semi-tolerance zones (based on the assumption that the process standard deviation of each part has an increasing linear relationship with its tolerance) being proposed in this research concerns the fidelity of the conditions of the product. Therefore, the approaches used for optimization avoid the assumptions and the disadvantages of the current models. For example, it independently optimizes the lower and the upper semi-tolerance zones, whereas some of the current models determine the entire tolerances, and then set the semi-tolerance zones for both sides equal to the smaller semi-tolerance zones or equal to the halves of tolerances determined from those models. Moreover, the model can be applied independently to any one of the inspection strategies for every part: non-inspection strategy, 100% inspection without reworking strategy or 100% inspection with imperfect reworking strategy. Another goal of this research is to develop a model that can optimize semi-tolerance zones of a product with either the minimum requirements for process capability indices or the specified minimum proportions of conforming units for the parts. Finally, we seek to optimize tolerances in a practical way, using commonly available off-the-shelf software.

Research Approaches

Optimizing semi-tolerance zones for the model being proposed in this research needs nonlinear programming because its objective function consists of fourth-order polynomials for conversion costs and quadratic functions for quality losses. Its non-linear constraints are associated with (1) the process standard deviation of the gap of the finished product resulting from the square root of the summation of the parts and the envelope variances, (2) the minimum requirements for process capability indices, and (3) the probability density function for the specified minimum proportion of conforming units for each side of the product.

Developing a specialized solution algorithm for the non-linear program is not the purpose of this research; the model is solved by using genetic algorithm in Evolver add-in for Microsoft Excel. Because of solving by the genetic algorithm (GA) in Evolver, the feasible solutions of the model are sequentially improved and are nearer to the global optimum semi-tolerance zones than those solved by Microsoft Excel Solver. Finally, experimental design is applied to analyzing the sensitivities of the total cost to the cost coefficients and constraints.

CHAPTER 2

LITERATURE REVIEW

Tolerance Design

Many papers have dealt with tolerance design since 1970's. Wu et al. (1988) surveyed, and then classified research in the field of tolerance analysis, tolerance allocation as follows. The models for tolerance analysis are worst-case, statistical, Spott's modified, modified statistical, mean shift, Monte Carlo model, moment, and hybrid models. The approaches used to solve tolerance allocation are proportional scaling, constant precision factor, Lagrange multiplier, geometric programming, linear programming, and the non-linear programming methods. The distributions of component tolerances were uniform, normal, truncated normal, and Weibull distributions. Zhang and Huq (1992) surveyed and classified more than 50 papers into five categories: (1) a dimensional tolerance chain technique, (2) geometric tolerances, (3) statistical and probabilistic methods used in tolerancing, (4) tolerance analysis and allocation, and (5) tolerances based on cost-tolerance algorithms. The dimensional tolerance chain technique was applied to a product where at least one common component's tolerance was shared in more than one tolerance chain. Most of papers dealing with geometric tolerancing have been published especially during the second half of 1990's. This research surveyed papers dealing with optimizing tolerances by considering: (1) decision variables, (2) tolerance-cost functions, (3) quality loss functions with or without truncation(s), (4) constraints and sensitivity analysis, (5) assumptions, and (5) approaches of the models.

The rest of this section discusses examples of the papers that have been studied in this research.

Decision Variables

Michael and Siddall (1981) optimized both design parameters and tolerances for the reciprocal power and exponential hybrid cost-tolerance model introduced by them. Chen et al. (1984) proposed a method for selecting optimal target values satisfying the allowable maximum tolerances while maintaining the performance standards by using interactive linear-programming based design algorithm. Dodson (1993) optimized the target values having the minimum total cost subject to the specified upper and lower limits. Nagarwala et al. (1994) simultaneously optimized tolerance allocation and process selection. Maghsoodloo (1995) optimized the unbalanced tolerances and the process means instead of the target values for a problem with asymmetrical quality loss functions based on the assumption that the process variations remained in control when the means of the processes were changed. Chen (1996) determined tolerances and non-independent dimensions for a tolerance allocation. Feng and Kusiak (1997) optimized tolerances and manufacturing processes using stochastic integer programming approach.

Tolerance-Cost Functions

Cost of tolerance is defined as the expense needed to achieve a certain level of dimensional accuracy. It depends on design and manufacturing. A design requiring tighter tolerance has higher cost. Manufacturing cost of a process with a small tolerance range is expensive. For instance, the machining costs of different operations such as grinding, milling, turning, honing and chamfering depend on the diameter and the length of the workpiece, cutting velocity, feed and so forth as can be seen in data from Machinability Data Center (1980). Bjorke (1978) suggested that the tolerance cost should include the actual time taken to produce and/or calibrate gages, tools, fixture, and time for inspection, along with the overhead.

Many types of cost functions for tolerance models were introduced and some cost functions were combined for better model with less fitting error. Generally, the higher descriptive cost model gives more accuracy of the cost, but it is more difficult to solve. Bennett and Gupta (1970) introduced the manufacturing cost as a power function of the tolerance. Speckhart (1972) introduced the exponential cost-tolerance model and determined the close-form solution of the cost model. Spotts (1973) introduced a reciprocal squared cost-tolerance model, and determined its close-form solution. Sutherland and Roth (1975) introduced reciprocal power cost-tolerance model along with its close-form solution. Trucks (1976) proposed empirical cost-tolerance data of frequently used production processes, and suggested that each tolerance variable should be bounded in order to avoid infeasible solutions in the optimization. Michael and Siddall (1981) introduced reciprocal power and exponential hybrid cost-tolerance model, and optimized both design parameters and tolerances. Dieter (1983) collected empirical cost-tolerance data of frequently used production processes. Chase and Greenwood (1988) introduced the reciprocal cost-tolerance model.

Wu et al. (1988) studied and compared the fitting errors of the existing continuous cost-tolerance models based on a general empirical cost-tolerance curve collected by Dieter. The conclusion was that the reciprocal power and exponential hybrid model has minimum modeling error and the exponential model was the second best. In addition, all five models (the reciprocal powers, the exponential, the reciprocal power and exponential hybrid, the reciprocal squared, and the reciprocal models) presented quite good approximations to the empirical data curve at the loose tolerance zone (≥ 3.5 mm), but had large fitting errors at the tight tolerance zone (≤ 0.1 mm). However, the exponential model has been widely used due to its simple form. They also suggested that the distribution of the resultant dimension is a normal distribution regardless of the distributions of the components when the number of the components is larger than five. Moreover, Ostwald and Blake (1989) compared and concluded that the cost model of Bennet and Gupta performed the best compared with those of Speckhart, Chase and Greenwood, and Spotts. They also introduced formula to determine the coefficients of the cost model of Bennet and Gupta for turning and that for boring.

Lee and Woo (1989) introduced a discrete cost-tolerance model and a tolerance optimization method using a reliability index and integer programming. Dong and Soom (1991) included the allowable tolerance ranges to the exponential cost-tolerance model with multiple dimensional chains. Cagan and Kurfess (1992) approximated manufacturing costs with hyperbolic functions for optimizing tolerance allocation over multiple manufacturing alternatives. Jeang (1993) optimized tolerance allocation based on the minimum total cost consisting of reworking cost from imperfect reworking strategy, scrap cost, quality loss and manufacturing cost as a reciprocal power function.

Dong et al. (1994) introduced six new production cost-tolerance models: (1) combined reciprocal power and exponential function, (2) combined linear and exponential function, (3) B-Spline curve, (4) cubic polynomial, (5) fourth order polynomial, and (6) fifth order polynomial. The new cost-tolerance models were compared with (1) exponential function introduced by Speckhart, (2) reciprocal square function introduced by Spotts, (3) reciprocal power function introduced by Sutherland and Roth, (4) reciprocal power and exponential hybrid function introduced by Michael and Siddall, (5) reciprocal power introduced by Chase and Greenwood, (6) discrete model introduced by Lee and Woo, and (7) exponential with an allowable range function introduced by Dong and Soom. Those cost functions were compared using the following eight empirical production cost-tolerance data curves: (1) general relation for frequently used production processes studied by Dieter (1983), (2) die casting, (3) investment casting, (4) true position of holes, (5) face milling, (6) turning on lathe, (7) rotary surface grinding, and (8) internal grinding. The models' parameters were determined using least square approximations. The comparison is performed over the entire valid tolerance zones of the production processes: tight, medium and loose tolerance zones (those are $\leq$ 0.1 mm, 0.1-0.35 mm and $\geq$ 0.35 mm, respectively). Kusiak and Feng (1995) and Feng (1995) included setup, inventory and scrap costs to the manufacturing cost for optimizing tolerance allocation.

Quality Loss Functions With Or Without Truncation(s)

Bisgaard et al. (1984) considered a linear quality loss function for a tolerance model with an assumption that the product's price was linearly reduced when the quality characteristic was less than the lower specification limit. Tang (1988) proposed a model for optimizing the specifications for a product with step loss functions and 100% inspection strategies. Taguchi (1984) popularized use of the quadratic quality loss function. Maghsoodloo (1992) suggested that the repair cost to the customer or the warranty cost to the producer as well as the producer's loss of market share should be included in quality loss. Li and Maghsoodloo (1995) mentioned that most papers in the field of tolerance design chose the smaller tolerances as the tolerances for both the upper and the lower sides, or set the process means at the middle points of the tolerances for the cases of unbalance tolerances. Feng and Kusiak (1996) mentioned that the quality loss is a small portion of total cost consisting of manufacturing cost and quality loss.

Kapur (1988) dealt with developing the total cost for optimizing symmetrical two-sided tolerance for only one part (component) with process mean equal to the nominal size without any constraint. He also proposed the total cost for optimizing tolerance for one part with process mean not equal to the nominal size without any constraint. The quality characteristic of the part was truncated at the specification limits that caused an asymmetrical truncation. The quality loss coefficients for the lower sides were equal to those for the upper sides for cases with the process means equal to and unequal to the nominal sizes. The objective functions of the models considered expected quality losses with truncated distributions whereas it considered expected scrap and reworking costs with non-truncated distributions

Kapur and Cho (1994) optimized product specifications considering quadratic quality loss functions with truncated Weibull distributions for for The-Smaller-The-Better and The-Larger-The-Better types of tolerances. Cho et al. (1996) applied singly truncated exponential distributions to the expected values of quadratic quality loss functions for optimizing the specifications of a product with 100% inspection strategy. Jeang (1997) applied truncated normal distributions to the quality characteristics shipped to the customer. Asymmetrical quality loss functions for all three types of tolerances (The-Nominal-The-Best, The-Larger-the-Better, and The-Smaller-The-Better types) were studied in the model. Cho and Phillips (1998) applied singly truncated Gamma distributions to quadratic quality loss functions for 100% inspection strategies to optimize product specifications.

Chen and Kapur (1989) considered the bias and the variance of the interaction between the characteristics for a multivariate quality loss problem. Raiman and Case (1990) discussed quadratic quality loss functions for a problem with multiple quality characteristics. The total quality loss could be obtained by adding the loss caused by each quality characteristic. Soderberg (1994) optimized target values for tolerance allocation considering both manufacturing costs and asymmetrical quality losses. The optimum target values should be moved in directions away from the more sensitive sides for the quality losses. Li and Maghsoodloo (1995) used first and second derivative to determine the process means for the cases with asymmetrical linear and asymmetrical quadratic quality loss functions. Wu et al. (1998) proposed models for tolerance allocation considering manufacturing costs along with symmetric or asymmetric quality loss functions for the worst case and the root sum square methods. They concluded that the quality loss and the manufacturing cost should be treated as equally important for tolerance designs. Bernardo and Saraiva (1998) simultaneously considered (1) deterministic instrument cost, (2) stochastic operating cost, (3) stochastic quadratic quality loss, and (4) stochastic control cost growing as the tolerance becoming smaller.

Constraints and Sensitivity Analyses

Springer (1951) discussed process mean optimization that was based on the maximum expected profit subject to the given product specification limits. The unit manufacturing cost was assumed as a linear function of the level of the quality having a normal distribution with a known variance. Speckhart (1972) considered constraints associated with the sums of the component tolerances less than or equal to the allowable assembly tolerances for the deterministic and the probabilistic tolerance allocation. Ostwald and Huang (1977) included setup constraints and the process shifts to optimizing tolerance allocation. Feng (1995), and Feng and Kusiak (1997) considered process shifts and setup constraints for optimizing tolerance allocation in order to ensure the satisfaction for the minimum required levels of manufacturing yields. The weighted statistical tolerance stack up was applied to the model.

Mishra and Rao (1982) considered tolerances as random variables with normal distributions and applied a chance constrained programming technique to optimize tolerance allocation. Lee and Woo (1990) included constraints of functionality and interchangeability to a probabilistic tolerance allocation model. Feng (1995) considered manufacturing variations as a type of constraint for probabilistic tolerance allocation. The manufacturing yield, the tolerance and the manufacturing cost for each component were linked together.

Chase and Greenwood (1988) proposed a model for optimizing tolerance allocation subject to either a constraint associated with the mean assembly tolerance requirement or a constraint associated with the variance of the assembly distribution. Kapur and Cho (1994) optimized product specifications subject to (1) constraints showing the function of the mean and that of the variance of a quality characteristic in terms of scale and shape parameters of a Weibull distribution, (2) constraints of truncated cumulative probabilities, and (3) the conditions on the numbers of standard deviations truncated. Wu and Tang (1998) mentioned that the nominal values of the functional characteristics specified by design engineers were viewed as the manufacturing targets of products. In addition, the assembly has to satisfy the functional requirement. Wu et al. (1998) proposed a model for optimizing tolerance allocation subject to constraints associated with the allowable minimum tolerances due to the capabilities of the machines and the allowable maximum tolerances due to the functional requirements. Li et al. (1998) proposed a model for robust tolerance allocation with multivariate normal distributions using stochastic programming. Its allowable maximum manufacturing costs were functions of the standard deviations. Functions of C_p, process capability indices, were substituted for the standard deviations in the constraints associated with the allowable maximum manufacturing costs. In addition, sensitivity analyses of the manufacturing costs determined from the model were performed.

Jeang (1995) minimized tolerance allocation based on the minimum total cost consisting of quality losses and manufacturing costs for multiple-characteristic parts. Jeang also suggested that not only the allowable ranges for the component variances along with those for the component tolerances, but also the effects of the component variances on the resultant variance along with the effects of the component tolerances on the resultant tolerance should be considered in the tolerance model. Cheng and Maghsoodloo (1995) analyzed the effects of the variations of the process means and those of the process variances for optimizing tolerance allocation.

Assumptions

Springer (1951) assumed that the unit manufacturing cost was a linear function of the level of the quality having a normally distribution with a known variance. The purpose of the model was to optimize process means subject to the given product specification limits. Wu et al. (1988) suggested that both Beta and normal distributions will obtain close approximation accuracy with a product consisting of more than five components. They also suggested that, in practice, when the number of components is larger than five, the resultant dimension is normally distributed regardless of the distributions of the components. Tang (1988) mentioned that, usually, the means of The-Nominal-The-Best type of characteristics could be approximately adjusted to their target values with low costs. Tang and Tang (1989) optimized specification limits for multiple quality characteristics for perfect inspection strategies based on the assumption that the quality characteristics were independent. Li and Maghsoodloo (1995) assumed that process variances remained in control when the process means changed for optimizing unbalance tolerance allocation with asymmetrical quality loss functions. Feng (1995) assumed that the processes producing components were independent resulting in the components' dimensions being independent.

Approaches

Wu et al. (1988) discussed mathematical procedures such as the proportional tolerance scaling and the constant precision factor functions. Lee and Woo (1990) developed a model for tolerance allocation considering the probability of the reliable region that was computed using a reliability index. The reliable region was the intersection of the tolerance region and the safe region for each dimension. Williams and Hawkins (1996) considered type I and type II errors from measurement. Wei (1998) considered the manufacturing, the scrap and the reworking costs, along with the process capability of the machine. The manufacturing costs were reciprocal square functions of tolerances those were modified to the functions of C_p. Moskowitz et al. (1999) minimized the maximum total cost for multivariate tolerance design where values of the process means and process variances were known, but their distributions were not known.

Speckhart (1972) utilized Lagrange multiplier method for minimizing exponential cost functions subject to nonlinear tolerance stackup constraints for both deterministic and stochastic cases. Dresner and Barkan (1993) optimized tolerance allocation having a single tolerance stackup or multiple tolerance stackups, which shared one or more common tolerance(s), by using numerical Lagrange multiplier method. Rajasekera and Fang (1995) optimized tolerance allocation, which had manufacturing exponential cost functions, by using Kuhn-Tucker necessary condition. Chen (1995) utilized Kuhn-Tucker necessary condition and Lagrange multiplier method for optimizing tolerance allocation with nonlinear multiple constraints. Chen (1996) applied Lagrange multiplier method to evaluate the efficiencies of the optimal tolerances for a product whose components' dimensions could be selected (dependent dimensions).

Sinha and Zoltners (1979) rapidly solved the multiple-choice knapsack model through setting some rules for choosing the non-zero variables. Candidates from the list were selected based on a last-in-first-out policy. Balakrishnan (1993) used a multiple-choice Knapsack model to optimize tolerance allocation for a product with alternative processes. Chase et al. (1990) applied (1) the zero-one search, (2) the exhaustive search, (3) the univariate search and (4) the sequential quadratic programming methods to optimize tolerance allocation. The univariate search and the sequential quadratic programming methods cannot guarantee that the global minimums will always be found. The exhaustive search method can always find the global minimums, but it is impractical for the model having more than 20-25 processes. Li et al. (1998) applied sequential quadratic programming algorithm embedded with Monte Carlo simulation to optimize tolerance allocation having multivariate normal distributions. The allowable maximum manufacturing costs were functions of standard deviations, which were substituted with the functions of C_p.

Cagan and Kurfess (1992) used simulated annealing algorithm and Monte Carlo simulation to optimize tolerance allocation over multiple manufacturing alternatives. Lin et al. (1997) utilized Monte Carlo simulation for allocating the tolerances for multi-dimensional chains. Zhang and Wang (1998) used simulated annealing algorithm to optimize tolerance allocation where the variance and the mean shift were considered as the variation of each component. Lee and Johnson (1993) utilized genetic algorithm and truncated Monte Carlo simulation for optimizing tolerance allocation. Kopardekar et al. (1993) utilized a neural networks approach instead of linear programming or Lagrange multiplier method for optimizing tolerance allocation for a product with different mean shifts and different distributions of parts' dimensions. Kapur and Cho (1994) applied a one-dimensional search procedure to optimize the specifications of a product having Weibull distributions. Laurent expansion was utilized for a quality loss function of The-Larger-The-Better type of a quality characteristic. Finally, Taylor's series approximation was applied to determine the expected quality loss. Nurre and Vedati (1998) suggested that tolerance allocation required consideration of stack-up conditions, machining constraints, scraps, and the sequence of the production operations. They utilized a gradient search technique for determining the direction of a function's steepest slope.

Feng (1995), and Kusiak and Feng (1995) applied Lagrange multiplier method to minimize the cost functions subject to nonlinear tolerance stackup constraints. Geometric programming was utilized for minimizing the manufacturing cost as an exponential model, and a reciprocal square model. Zero-one integer programming was used for a problem having alternatives for processes. Moreover, multiple-choice 0-1 knapsack model was used for a multidimensional problem. They concluded that the integer programming approach is suitable for solving linear deterministic problems. Design of experiment (DOE) approach can be used in both linear and nonlinear cases, but it is more appropriate for nonlinear problems, and it can be used to solve probabilistic problems. Taguchi method can be applied to probabilistic problems as well, but its solutions have higher costs than those from DOE. Jeang (1995) applied zero-one integer programming to a product with discrete manufacturing costs. Feng and Kusiak (1997) used stochastic integer programming approach to determined tolerances and select manufacturing processes. The process mean shifts and the weighted statistical tolerance stackups were considered in the model.

Chen et al. (1984) used interactive linear-programming based design algorithm to determine optimum nominal values subject to the allowable maximum tolerances. Dupinet and Balazinski (1994) utilized a hybrid technique by using fuzzy logic principles for solving tolerance allocation. The proportional scaling and the constant precision factor methods were applied to the model. Bare et al. (1996) used first-order approximation for the assembly variance in order to relate it to the component variances. Geometric programming was used to solve a problem whose components' costs were reciprocal functions with unequal exponents. A descent method was applied to solve the problem with negative exponential cost functions. Wei and Lee (1997) formulated the process capability of the equipment to assure that the tolerances allocated were producible. Nonlinear programming was applied to solve the model. Bernardo and Saraiva (1998) proposed robust optimization for process parameters and tolerance design. Hermersley sequence sampling technique developed by Diwekar and Kalagnanam (1996, 1997a and 1997b) was utilized for reducing the number of observations, but so that the result was still reliable. They also used a subroutine allowing considering several types of input probability density functions, which are independent or correlated, and continuous or discrete.

Truncated Distributions

The probability density function, the expected value and the variance of a doubly truncated normal distribution are found in *Continuous Univariate Distribution Volume 1* by Johnson and Kotz (1970). These expressions are useful in models involving scrap and rework, where the original process distribution may become truncated.

Process Capability Indices

A process capability index is used to measure the ability of a process with respect to the specification limits, the specification limit and the process mean, or the specification limit and the target value. C_p can give information about the ratio between the tolerance range and the process standard deviation. It does not consider the location of the mean. As a result, this index cannot give sufficient information about the process capability when the process mean and the target value are not at the middle point of the tolerance. C_{pk} for the case with the target value being at the midpoint of the specification interval was introduced by Kane (1986). It measures the process capability with respect to the process mean and the specification limit on the side giving the smaller value. C_{pm} for the case with the target value being at the midpoint of the specification interval was introduced by Chan et al. (1988). It compares the square root of the sum of the process variance and the square of the deviation of the process mean from the target value to the tolerance range. It should not be used to measure the capability of the process that does not have a target value equal to the midpoint of the specification interval. This results from the index having the same value whether the process mean shifts to the right or the left side of the target value, although those process means have different proportions of conforming units. Later, C^{*}_{pm} was defined by substituting the tolerance range in the numerator of C_{pm} with the distance between the target value and the specification limit on the side having the shorter distance. As a result, it can measure the capability of the process only for the narrower side, not the capability of the entire tolerance range. C_{pmk} was introduced by Pearn et al. (1992). $C_{pmk} = \frac{d - |\mu - m|}{3\sqrt{\sigma^2 + (\mu - T)^2}}$, where d = the half distance of the tolerance range, μ = the process mean, σ^2 = the process variance, m = the middle point between the lower and the upper specification limits, and T = the target value. However, it cannot measure the actual process capability for the entire range of the limits. The above indices are developed for measuring the capabilities of normally distributed processes with target values being at the middle points of the specification intervals.

There are some interesting process capability indices for unbalanced and/or non-normal tolerances. In addition to proposing C_{pk}, Kane (1986) also introduced an index for asymmetric tolerance, $C^*_{pk} = \frac{d - |T-m| - |\mu - T|}{3\sigma}$. Similarly, supplementing their C_{pm}, Chan et al. (1988) introduced an index for asymmetric tolerance, $C^*_{pm} = \frac{d - |T-m|}{3\sqrt{\sigma^2 + (\mu - T)^2}}$. Vannman (1995) introduced $C_p(u, v) = \frac{d - u|\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}$, where u and v are equal to or greater than 0 and proposed the simplified index with u = 1, $C_p(v) = \frac{d - |\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}$, in addition to suggesting the more complex process capability index $C_{pa}(u,v) = \frac{d - |\mu - m| - u|\mu - T|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}$. Johnson et al. (1994) introduced $C_{jkp} = \frac{1}{3\sqrt{2}} Min\left[\frac{USL - T}{\sqrt{E_{X>T}[(X-T)^2]}}, \frac{T - LSL}{\sqrt{E_{X<T}[(X-T)^2]}}\right]$. Bai and Choi (1997) applied the weighted variance method to measure the degrees of skewness found in non-normal process data for some process capability indices, one example being

$$C^{*w}_{pm} = \min\left[\frac{USL - T}{3\sqrt{\sigma^2 + (\mu - T)^2}\sqrt{2\Pr(X \le T)}}, \frac{T - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}\sqrt{2(1 - \Pr(X \le T))}}\right].$$

Another approach for measuring the capability of a non-normal process is transforming the original data into a normal distribution, and then choosing an appropriate index for the normal distribution.

CHAPTER 3

MODEL DEVELOPMENT

Modeling Approach Goals

Each part of products in the context of optimizing tolerance allocation has the following characteristics:

(1) the process mean can be either equal or unequal to the nominal size,

(2) there is possible asymmetry in the upper and the lower

- quality loss coefficients
- specified minimum requirements for the process capability indices
- specified minimum and maximum semi-tolerance zones
- specified minimum proportions of conforming units of the dimension resulting from assembling every part into the envelope, and

(3) there are three options to be chosen among: non-inspection strategy (NI), 100% inspection without reworking strategy (IWR) or 100% inspection with imperfect reworking strategy (IIR).

Generally, current models assume that the process means can be adjusted to equal the nominal sizes specified from parameter designs, but this may be economically impractical. All current models consider the specified minimum proportions of conforming units in constraints associated with the specified minimum quality criteria.

They do not consider the specified minimum requirements for process capability indices, which are sometimes required by customers, in the constraints. The current models contain many assumptions leading to oversimplification. For instance, they (1) assume a normal distribution for a part with skewed distribution, or approximate the lower side of the skewed distribution by one normal distribution and approximate the upper side by another normal distribution, (2) use generic distribution, (3) assume that the optimum semi-tolerance zones for the upper and the lower sides are equal, or (4) set both semi-tolerance zones at the value for the narrower side. Due to the conditions and limitations of the current models, the semi-tolerance zones determined from the current models are not the optimum solutions for real problems. The model being proposed in this research can independently optimize the upper and the lower semi-tolerance zones, and can eliminate the above disadvantages and limitations (except for the skewed distribution) of the current models for optimizing tolerance allocation. Therefore, this research deals with developing a model for optimizing tolerance allocation (Model OTA) based on the criterion of minimum total cost.

Application Scopes for Model OTA

While not to be used for geometric tolerance or tolerance analysis problems, model OTA can be applied to linear dimensional tolerance allocation problems, specifically two-sided tolerance problems with normal distributions. The conventional formulas of the expected value and the variance of a truncated normal distribution are used to calculate the expected quality loss for a normal distribution.

In addition, any one of three inspection strategies --non inspection, 100% inspection without reworking, and 100% inspection with imperfect reworking-- is chosen independently for each part. For the non-inspection strategy (NI), the finished units of a part are not measured or gauged in relation to the specification limits; they are used in the assembly without inspection. For 100% inspection without reworking strategy (IWR), the finished units of a part are measured or gauged in order to compare the produced dimensions with the specification limits to determine conformities. Those units with the dimensions within the specification limits are accepted for the assembly; otherwise they are scrapped. For 100% inspection with imperfect reworking strategy (IIR), the finished units of a part are measured or gauged against the standard. The units with the dimensions within the specification limits are accepted for the assembly whereas those with the dimensions less than the lower specification limit are rejected as scrap, and those with the dimensions greater than the upper specification limit are reworked using the same manufacturing process to produce new units. The reworked output is accepted, scrapped, or reworked again; hence, the label is imperfect reworking. Since the same manufacturing process is used, the proportion of each type of the output is the same for reworked units as for new.

Conditions and Assumptions for Model OTA

Model OTA can be applied to a problem corresponding to the above situations along with conditions and assumptions listed below:

(1) The nominal size of the dimension for each part is fixed at the specified value.

(2) The process variance for each of appropriate settings of the operation speeds of a machine has been determined.

(3) The process mean of the dimension for each part is already adjusted to the economic value where the total saving minus total increasing cost due to

adjustment is the greatest. The economic value of the process mean may differ from the nominal size. The process mean is stable over the practical range of the operation speeds of a machine. Since the value of the process standard deviation depends on the setting of the operation speeds, the process mean is not changed for the entire allowable range for the process variance.

(4) There is no inspection error.

(5) The specified minimum proportions of conforming units for below and above the nominal size for the product are measured as the numbers of standard deviations in the semi-tolerance zones of the gap. And then, they are transformed to the values of the proportions of conforming units based on a normal distribution, which corresponds to the chosen inspection strategies, with the process mean equals that of the envelope minus those of all parts, and the process variance equals the summation of the process variances of the envelope and the parts.

(6) For each part, one can independently chose any one of the inspection strategies.

(7) Users have to choose the inspection strategy for each part, the envelope and the gap before applying model OTA to optimize the semi-tolerance zones. Only one inspection strategy can be applied to each part.

(8) The specified minimum proportion of conformity for each side of each part or the product is the maximum value chosen from all of the values requested from all sources or all conditions.

(9) The specified minimum semi-tolerance zone for each side of each part is the maximum value chosen from all of the values specified by all sources or all conditions.

Before considering the modeling approach in detail, brief information about the objective function and the constraints of model OTA need to be discussed.

The goal of the objective function of model OTA is to minimize the total cost that is a function of the semi-tolerance zones and the process variances for all parts. Each inspection strategy affects the total cost and the constraints associated with the specified minimum proportion of conforming units for below and above the nominal size of the product. The total cost of the objective function for 100% inspection with imperfect reworking strategy, which has the most types of costs, consists of (1) conversion cost, (2) expected inspection cost, (3) expected scrap cost, (4) expected reworking cost, and (5) expected quality loss. Since the context of the model is routine production, manufacturing a sequence of the product over a long period of time, the expected values for last four costs are used instead of their values for each unit. Every type of cost in this research is in cost per unit. Based on conventional methods, the expected inspection, scrap and reworking costs are each measured as a percent of the conversion cost. Each cost is defined as follows:

(1) Conversion cost is all costs chargeable for the production of a product, except material, inspection, reworking, and scrap costs.

(2) Inspection cost is all expenses involved in measuring or gauging the quality of a unit and comparing it with the specified requirements to determine the level of conformity of the finished unit after the final manufacturing process.

(3) Since the scrap can be recycled or sold, the manufacturer can receive income from the scrap. However, there is some expense for all of these processes. Therefore, the scrap cost includes the total expense, including the material cost, minus this income.

(4) Reworking cost is the expense needed to correct the defects or deficiencies of a unit so that it meets the requirements for the finished unit.

(5) Quality loss includes the costs passed on to the customer or society until it is no longer useable. It can be losses due to harmful effects, pollution, operating cost, or other losses. In addition, it includes all costs to the customer or society due to the improper functioning of a part when the actual quality characteristic not meeting the target value. For this research, the quality loss, therefore, is zero when the part's dimension equals the nominal size.

Since each inspection strategy has at least one type of costs different from the others, it is necessary to consider costs for each. All types of costs for each inspection strategy are listed below:

(1) the conversion cost and the expected quality loss for NI

(2) the conversion, the expected inspection, and the expected scrap costs, along with the expected quality loss for IWR, and

(3) the conversion, the expected inspection, the expected scrap and the expected reworking costs, along with the expected quality loss for IIR.

This research deals with developing a model for optimizing tolerance allocation based on the assumption that the process standard deviation for each part has an increasing linear relationship with the tolerance. The constraints of model OTA are listed below:

(1) the constraint associated with the process variances of the parts to be assembled into the envelope (whose variance is assumed fixed) in order to satisfy the allowable maximum gap standard deviation.

(2) constraints associated with the minimum requirements for the quality criteria, which are

a. the specified minimum requirement for the process capability index for each side of each part,

b. the specified minimum proportion of conformity for each side of the product (the gap, in this research)

(3) the following four constraints associated with each side of each part,

a. the allowable maximum semi-tolerance zone

b. the allowable minimum semi-tolerance zone

c. the allowable maximum process standard deviation

d. the allowable minimum process standard deviation

The rest of this chapter describes the approach for developing model OTA in detail, beginning with the approach for developing the objective function and the constraints before concluding with the entire model. For notation convenience, the subscript i for each symbol for each of *m* part that has a potentially different characteristic is suppressed unless necessary for understanding.

Approaches for Developing Objective Function

Below is the method used for developing the objective function of the model OTA for the non-inspection, the 100% inspection without reworking, and the 100% inspection with imperfect reworking strategies.

Objective Function

The purpose of optimizing tolerance allocation in this research is to optimize semi-tolerance zones in order to minimize total cost of the product based on the assumption that the process standard deviation of each part has an increasing linear relationship with the tolerance. Since each strategy of inspection has differences in the expected inspection, reworking and scrap costs, and in the expected quality loss, the grand total cost has to be evaluated in order to choose the optimal strategy. The following sections detail each of the costs for producing one unit of each part for each strategy.

Non-Inspection Strategy (NI)

Conversion cost

While several researchers proposed production cost-tolerance models, those proposed by Dong et al. (1994) have smaller values for the fitting errors than other models. Moreover, their models were plotted and determined in terms of the percentages of production cost increases compared with the production cost of casting process vs. tolerances. That means their models can be applied widely to the products produced from manufacturers with differences in costs of materials, labors, operating, overhead and so on. (See Appendix A. for a full explanation of the Dong model.)

The production cost-tolerance model developed by Dong et al. is the cost due to the whole tolerance range. The purpose of this research is to develop a model for optimizing semi-tolerance zones for each part under the conditions: (1) the process mean may not be coincident with the nominal size, and (2) there is possible asymmetry in the upper and the lower quality loss coefficients, the upper and the lower minimum requirements for the process capability indices, and/or the upper and the lower specified minimum semi-tolerance zones. Since those situations make the optimum upper semi-tolerance zone not equal to the optimum lower semi-tolerance zone, the model needs to optimize the semi-tolerance zones for each side independently. As a result, the conversion cost needs to be divided into two components, one for each of the upper and the lower semi-tolerance zones. The concept for dividing the conversion cost into two components can be applied to a case with either mean offset from the nominal size or not. To aid in understanding this concept, a case where the process mean falls on the right side of the nominal size and the lower semi-tolerance zone is smaller than the upper is shown in Figure 3-1.

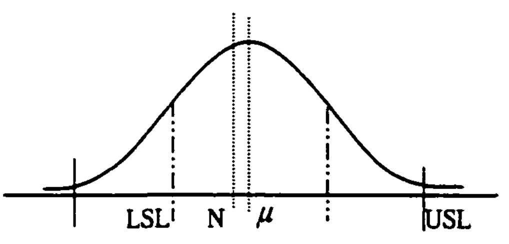


Figure 3-1. Concept for Dividing Conversion Cost into 2 Components for The Lower and Upper Sides

Each part has a potentially different normal distribution X_i for the produced dimension, but for notational convenience the subscript i will be suppressed unless required for clarity. Based on Figure 3.1, let

$E(X) = \mu$ = process mean of the produced dimension X for each independent part

$Var(X) = \sigma^2$ = variance of the produced dimension X for each independent part

N = nominal size (target value) for each part

$LSL = N - \Delta_L$ = lower specification limit

$USL = N + \Delta_U$ = upper specification limit

Δ_L = lower semi-tolerance zone for each part being optimized

Δ_U = upper semi-tolerance zone for each part being optimized

$$P_a = \int_{N-\Delta_L}^{N+\Delta_U} f_X(x)\,dx$$

= proportion of conformity for a new manufactured unit of each part

$$P_{aL} = \int_{N-\Delta_L}^{N} f_X(x)\,dx$$

= proportion of conformity below the nominal for a new manufactured unit of each part

$$P_{aU} = \int_{N}^{N+\Delta_U} f_X(x)\,dx$$

= proportion of conformity above the nominal for a new manufactured unit of each part

$$P_L = \int_{-\infty}^{N} f_X(x)\,dx$$

$$P_r = \int_{N+\Delta_U}^{\infty} f_X(x)\,dx$$

$$P_s = \int_{-\infty}^{N-\Delta_L} f_X(x)\,dx$$

$$P_U = \int_{N}^{\infty} f_X(x)\,dx$$

The concept used for dividing the conversion cost into two components is developed for only two-sided tolerance problems where the semi-tolerance zone on the smaller side is greater than one standard deviation. When the smaller semi-tolerance zone is only one standard deviation wide, the total conversion cost for both components calculated from this concept differs from the conversion cost calculated from Dong et al's model by approximately ten percent. Moreover, in practice, the optimum value of the smaller semi-tolerance zone would be far greater than one standard deviation, with corresponding improvement in accuracy of the approximation. The simple sum of the conversion costs calculated from the two semi-tolerance zones is very different from the cost calculated from Dong et al's model. To adjust this to approximate the cost from Dong et al's model, a procedure that recognizes the asymmetry of the tolerance is necessary.

In this procedure the conversion cost for each of the lower and the upper semi-tolerance zones is reflected as if each has symmetrical tolerance. Each cost component is then weighted by the proportion of conformity of that side from the total proportion of the conformity for both sides. Without multiplying each cost component by its proportion, the summation of the cost for both sides will be about two times the conversion cost calculated from Dong et al's model. This research uses a reflection point at the process mean instead of the nominal size because the produced dimension has a normal distribution that is symmetric about the mean rather than the nominal. The result is a conversion cost for the entire tolerance band that approximates the cost that would be found by Dong et al, but also recognizes the effect of asymmetrical tolerances with mean offset from nominal value.

For assigning one component of the conversion cost to the lower semi-tolerance zone, the process mean is used to calculate the conversion cost. The distance for calculating the conversion cost for the lower side is $\Delta_L^\bullet = \Delta_L + (\mu - N)$. Since $\Delta_L^\bullet$ is the distance for only one side of the tolerance, it is multiplied by 2 as if the part has the tolerance with balanced semi-tolerance zones for both sides. Then it has to be weighed by dividing the proportion of conformity for the lower semi-tolerance zone by the total proportion of conformity of that part. The weight for the conversion cost for the lower side of the nominal size is $\frac{P_{aL}}{P_a}$. Finally the conversion cost for the lower side of the nominal size for each part (see a full explanation of the Dong model in Appendix A) is:

$$C_{C_L} = \left[\left[\left\{A + 2B\Delta_L^\bullet + C\left\{2\Delta_L^\bullet\right\}^2 + D\left\{2\Delta_L^\bullet\right\}^3 + F\left\{2\Delta_L^\bullet\right\}^4\right\}\left\{\frac{P_{aL}}{P_a}\right\}\left\{\frac{1}{100}\right\} + 1\right]C_M \quad \text{\_\_(3-1)}$$

The calculation of the other component of the conversion cost for the upper semi-tolerance zone is the similar to that used for the lower. The distance for calculating the conversion cost for the upper side is $\Delta_U^\bullet = \Delta_U - (\mu - N)$. The weight for the conversion cost for the upper side of the nominal size is $\frac{P_{aU}}{P_a}$. Finally, the conversion cost for the upper side of the nominal size for each part is:

$$C_{C_U} = \left[\left[\left\{A + 2B\Delta_U^\bullet + C\left\{2\Delta_U^\bullet\right\}^2 + D\left\{2\Delta_U^\bullet\right\}^3 + F\left\{2\Delta_U^\bullet\right\}^4\right\}\left\{\frac{P_{aU}}{P_a}\right\}\left\{\frac{1}{100}\right\} + 1\right]C_M \quad \text{\_\_(3-2)}$$

Therefore, the total conversion cost for producing one unit of each part is:

$C_C = C_{C_L} + C_{C_U}$ __(3−3).

Although material costs for the product used in this study have not been included, they would need to be added if choosing different machine models would affect the material costs.

Expected Quality Loss

Traditionally, quality loss was determined using classical loss function (step loss function), which is based on the concept that there is no loss when the quality characteristic of the product is within the specification limits even though it does not equal the target value. Classical quality loss function is shown in Figure 3-2 below:

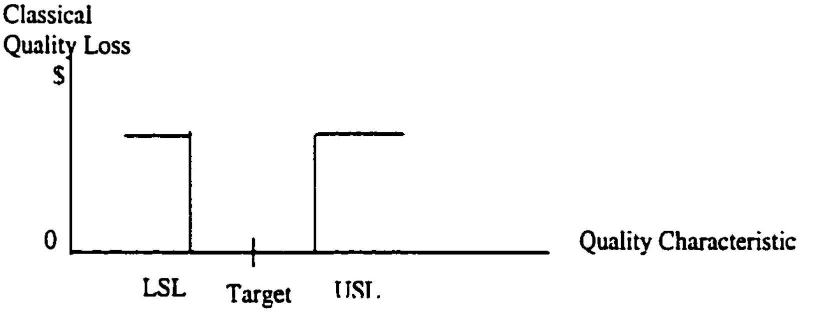


Figure 3-2. Classical Quality Loss

Quality loss, according to Taguchi et al. (1989), includes the costs passed on to the customer or society until it is no longer useable. It can be losses due to harmful effects, pollution, or other losses. This concept for quality loss includes all costs to the customer or society due to the improper functioning of a part when the actual quality characteristic not meeting the target value. The quadratic quality loss function is not complicated, and it is more practical than the classical loss function. As the purpose of this research is not to develop a quality loss function, the simple quadratic loss function will be used in the model of this research, not the classical or the polynomial loss function. Based on the above definition, the quality loss is assumed to be zero when the part's dimension equals the nominal size. The quadratic quality loss with symmetrical quality loss coefficients for the lower and the upper sides for each part for NI is shown in Figure 3-3, and its formula is:

$QL = K(x - N)^2 \quad for -\infty \le x \le \infty$ where

K = quality loss coefficient for each part (a constant)

x = dimension of each part.

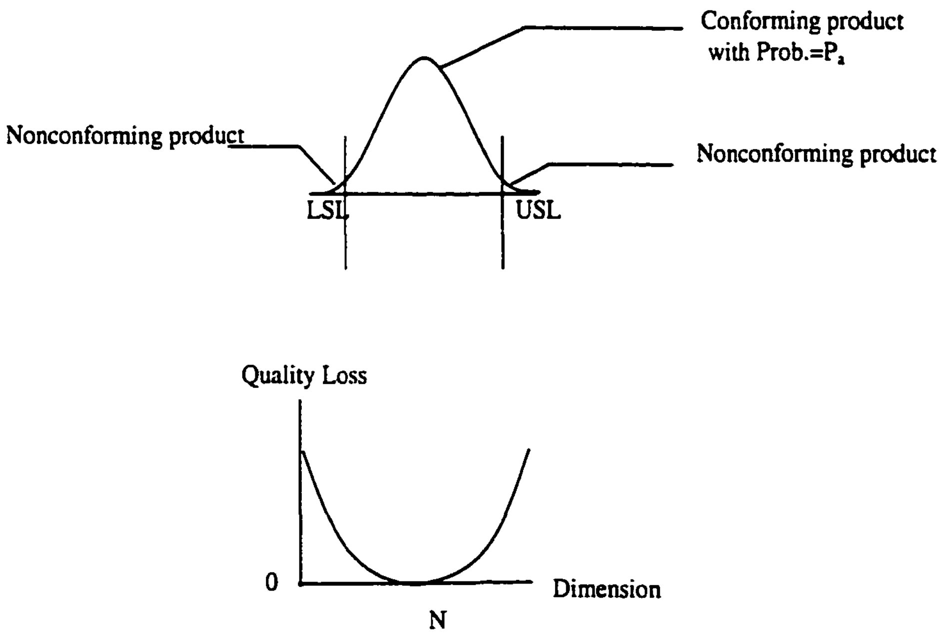


Figure 3-3. Distribution for the Part Dimension and Symmetrical Quality loss for NI

Since the context of the model is routine production, manufacturing a sequence of the product over a long period of time, the expected value of the quality loss is used instead of the loss for each unit. The expected value of the quality loss can be determined based on the properties of mathematical expectation shown below:

$$E\left[\left\{K(X-N)^2\right\}\right] = K\,E\left[X^2 - 2XN + N^2\right]$$

$$= K\left[E\left(X^2\right) - 2E(X)N + N^2\right]$$

$$= K\left[Var(X) + \{E(X)\}^2 - 2E(X)N + N^2\right]$$

$$= K\left[Var(X) + \{E(X) - N\}^2\right]$$

$$= K\left[\sigma^2 + \{\mu - N\}^2\right] \quad \_\_(3-4)$$

where

$Var(\bullet) = \sigma_\bullet^2$ = variance of its argument

$E(\bullet) = \mu_\bullet$ = expected value of its argument

The expected quality loss for each part can be also written as the integration formula below:

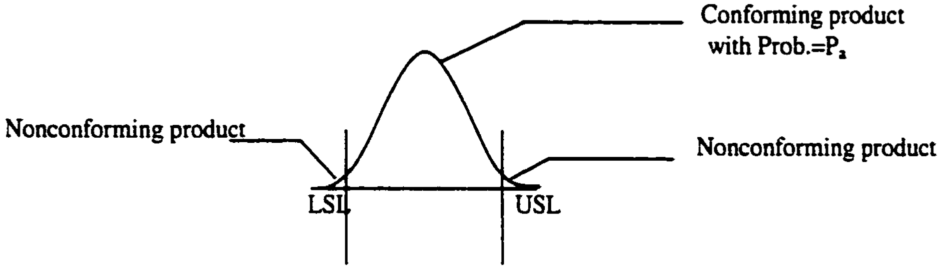
$$\int_{-\infty}^{\infty} K(x-N)^2 f_X(x)\,dx \quad \_\_(3-5)$$

Since the purpose of this research is to develop model OTA where the quality loss coefficients for the upper side may differ from that for the lower side, the quality loss with asymmetrical quality loss coefficients for each part for NI is shown in Figure 3-4, and is formulated below:

$$QL = \begin{cases} K_L(x-N)^2 & for -\infty < x \le N \\ K_U(x-N)^2 & for\ N \le x < \infty \end{cases} \quad \_\_(3-6)$$

where K_L = quality loss coefficient for lower side of each part (a constant)

K_U = quality loss coefficient for upper side of each part (a constant)



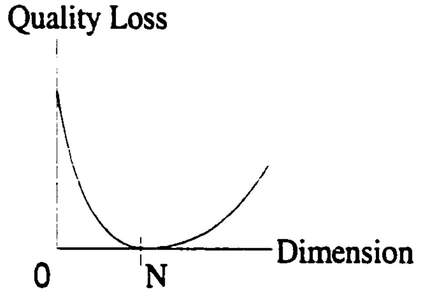


Figure 3-4. Distribution for the part dimension and Asymmetrical Quality Loss for NI

The expected quality loss with asymmetrical quality loss coefficients can also be expressed in terms of integration function below:

$$E(QL_{NI}) = \int_{-\infty}^{N} K_L (x-N)^2 f_X(x)dx + \int_{N}^{\infty} K_U (x-N)^2 f_X(x)dx \quad \text{\_\_(3-7)}$$

Since Microsoft Excel and Evolver, the software used in this research, cannot directly determine the expected quality loss formulated in equation (3-5) or (3-7) because they do not have an integration function, the formula of the expected quality loss must be written in terms of the expected value and of the variance of that normal distribution with truncation so Microsoft Excel can be used. The detail for transforming the expected quality loss in equation (3-7) to the form that is convenient to be calculated by using Microsoft Excel is expressed in Appendix B. Equation (3-7) can be written as:

$$E(QL_{NI}) = P_L K_L \int_{-\infty}^{N} (x-N)^2 f_{T_{S_L}}(x)dx + P_U K_U \int_{N}^{\infty} (x-N)^2 f_{T_{S_U}}(x)dx$$

where

T_{S_L} is a singly truncated normal distribution with the upper truncation point N , and T_{S_U} is a singly truncated normal distribution with the lower truncation point N.

From equation (3-4), the expected quality loss can be finally written as

$$E(QL_{NI}) = P_L K_L \left[\sigma^2_{T_{S_L}} + \left(\mu_{T_{S_L}} - N\right)^2\right] + P_U K_U \left[\sigma^2_{T_{S_U}} + \left(\mu_{T_{S_U}} - N\right)^2\right] \_\_(3-8)$$

The total cost for producing one unit of each part for NI can be expressed as

$$C_{NI} = \text{Conversion Cost} + \text{Expected Quality Loss}$$
$$= C_C + E(QL_{NI})$$

<u>100% Inspection Without Reworking Strategy (IWR)</u>

The second strategy is 100% inspection without reworking (IWR). Every unit is inspected for conformity after the final manufacturing process, but there is no reworking for IWR. Those units with dimensions within the specification limits are accepted for assembly; others are scrapped. Therefore, conversion cost, expected inspection and scrap costs, and expected quality loss are the cost components for this inspection strategy.

Conversion cost

Since conversion cost is not affected by the inspection strategy, equation (3-3) for NI can also be applied to IWR.

Expected Inspection Cost

The inspection cost for each inspected unit of each part is equal to F_I percent of the conversion cost for that part, $C_{I_{IWR}} = F_I C_C$. The expected value of the inspection cost for each manufactured unit equals the cost to inspect each unit because each is inspected for IWR. Therefore, the expected inspection cost for each manufactured unit is $E(C_{I_{IWR}}) = F_I C_C$.

Expected Scrap Cost

Since the scrap can be recycled or sold, the manufacturer may receive income from the scrap. However, there is some expense for all of these processes. Therefore, the scrap cost includes the total expense from the beginning until the end of producing a unit of nonconforming product minus this income. Moreover, it has to include the material cost because the conversion cost does not include the material cost. A scrapped unit of each part for the model is assumed equal to F_S percent of the conversion cost for that part. Since only the units with dimensions outside the specification limits are scrapped and the context for the model OTA is routine production, the expected value, not the individual value, of the scrap cost for each manufactured unit is used as shown below:

$$C_{S_{IWR}} = \begin{cases} F_S C_C & \text{for } -\infty < x < N - \Delta_L \text{ or } N + \Delta_U < x < \infty \\ 0 & \text{for } N - \Delta_L \le x \le N + \Delta_U \end{cases}$$

$$E(C_{S_{IWR}}) = F_S C_C \int_{-\infty}^{N-\Delta_L} f_X(x)\,dx + F_S C_C \int_{N+\Delta_U}^{\infty} f_X(x)\,dx$$

$$= F_S C_C \{P_s + P_r\}$$

Expected Quality Loss

As has been explained, only the conforming units, those accepted for the assembly process, can cause quality loss. The quality loss for each manufactured unit for IWR is

$$QL_{IWR} = \begin{cases} K_L * (x - N)^2 & \text{for } N - \Delta_L \le x \le N \\ K_U * (x - N)^2 & \text{for } N \le x \le N + \Delta_U \\ 0 & \text{otherwise} \end{cases}$$

and is shown in Figure 3-5.

The concept for developing the expected quality loss for IWR is the same as that for NI. However, the specification limits for below and above the nominal size used to determine the expected quality losses for IWR differ from those for NI. The normal distribution for the dimension of the manufactured units and the asymmetrical quadratic quality loss function for the model OTA for IWR for one unit of each part are shown in Figure 3-5 below:

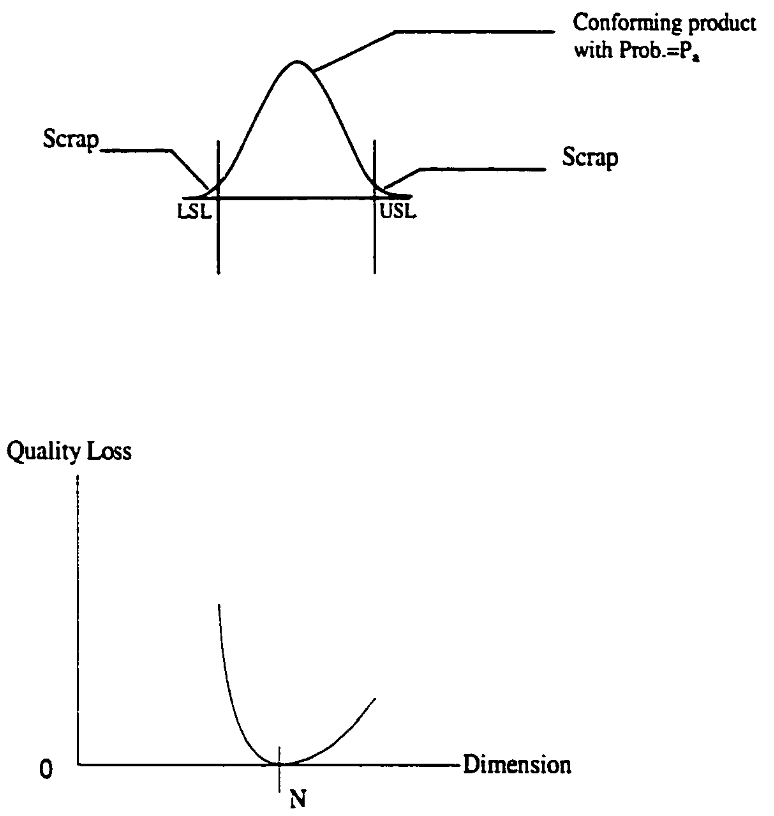


Figure 3-5. Distribution for the part dimension and Asymmetrical Quality Loss for IWR

Since quality loss occurs when the dimension of the conforming units, which is a doubly truncated normal distribution, is not equal to the nominal size, the expected quality loss for each part for IWR (see Appendix B) can be written as follow:

$$E(QL_{IWR}) = P_{aL}K_L\left[\sigma^2_{T_{D_L}} + \left(\mu_{T_{D_L}} - N\right)^2\right] + P_{aU}K_U\left[\sigma^2_{T_{D_U}} + \left(\mu_{T_{D_U}} - N\right)^2\right] \quad (3-9)$$

where T_{D_L} is a doubly truncated normal distribution with the lower truncation point $N - \Delta_L$ and the upper truncation point N, and T_{D_U} is a doubly truncated normal distribution with the lower truncation point N and the upper truncation point $N + \Delta_U$.

The total cost for producing one unit of each part for IWR can be written as:

$$
\begin{aligned}
C_{IWR} &= \text{Conversion cost} + \text{Expected Quality Loss} \\
&\quad + \text{Expected Inspection Cost} + \text{Expected Scrap Cost} \\
&= C_C + E(QL_{IWR}) + E(C_{I_{IWR}}) + E(C_{S_{IWR}})
\end{aligned}
$$

<u>100% Inspection With Imperfect Reworking Strategy (IIR)</u>

In the third strategy, 100% inspection with imperfect reworking, every unit is inspected for conformity after the final manufacturing process. The units with dimensions within the specification limits are accepted whereas those with the dimensions less than the lower specification limit are rejected as scrap and those with the dimensions greater than the upper specification limit are reworked using the same manufacturing process. Therefore, conversion cost, expected inspection, scrap, and reworking costs, and expected quality loss are the cost components for this inspection strategy. The distribution for the dimension of the manufactured units for the model OTA for IIR for each part is shown in Figure 3-6 below:

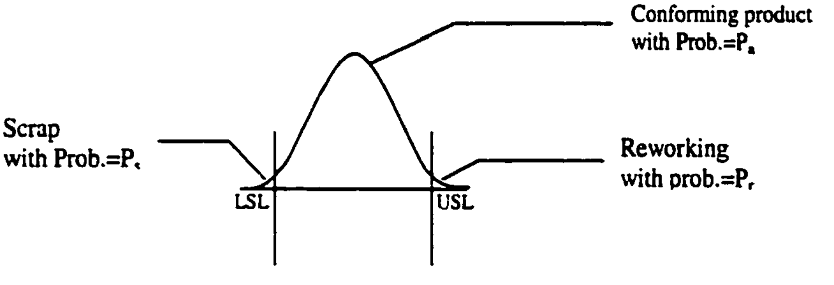


Figure 3-6 Distribution of the part dimension for IIR

Since the units with dimensions greater than the upper specification limit are reworked using the same manufacturing process to produce new units, the proportion of each type of the reworked output that is accepted, scrapped or reworked again is the same as for new units. The total number of each type per produced unit for each part is determined as follow:

$$\text{The number of inspected units per produced unit} = 1 + \begin{Bmatrix} P_r + P_r{*}P_r + P_r{*}P_r^{\,2} \\ + P_r{*}P_r^{\,3} \;+ \ldots \end{Bmatrix}$$

$$= \sum_{j=0}^{\infty} P_r^{\,j}$$

$$= \frac{1}{1\text{-}P_r}$$

$$\text{The number of scraped units per produced unit} = P_s \cdot (\text{the number of inspected units})$$

$$= \frac{P_s}{1\text{-}P_r}$$

$$\text{The number of reworked units per produced unit} = P_r \cdot (\text{the number of inspected units})$$

$$= \frac{P_r}{1\text{-}P_r}$$

$$\text{The number of conforming units per produced unit} = P_a \cdot (\text{the number of inspected units})$$

$$= \frac{P_a}{1\text{-}P_r}$$

Conversion cost

Since conversion cost is not affected by any inspection strategy in this research, the equation (3-3) for NI can be applied to that for IIR.

Expected Inspection Cost

The inspection cost for each inspected unit of each part is also assumed equal to F_I percent of the conversion cost for that part. Since every reworked unit is inspected the same as a new one, its inspection cost has been set at F_I percent as well. The expected inspection cost for each manufactured unit of each part for IIR including the effect of imperfect reworking is:

$$E(C_{I_{IIR}}) = F_I C_C \left\{ \frac{1}{1-P_r} \right\}.$$

Expected Scrap Cost

The scrap cost for each scrapped unit of each part is assumed equal to F_S percent of the conversion cost of that part, the same as that for IWR. The expected scrap cost for each manufactured unit for IIR including the effect of imperfect reworking is shown below:

$$E(C_{S_{IIR}}) = F_S C_C \frac{P_s}{1-P_r}.$$

Expected Reworking Cost

The reworking cost for each reworked unit of each part for the numerical examples in this research has been set at F_R percent of the conversion cost for that part. Since the units with dimensions greater than the upper specification limit are reworked again using the same manufacturing process to produce new units, the expected reworking cost for each manufactured unit of each part is shown below:

$$E(C_{R_{IIR}}) = F_R C_C \frac{P_r}{1-P_r}.$$

Expected Quality Loss

The concept for developing the expected quality loss for IIR is the same as that for IWR (see page 40-41). However, the loss for this strategy has to include the effect of imperfect reworking. The expected quality loss for each manufactured unit of each part for IIR including the effect of imperfect reworking can be written as:

$$E(QL_{IIR}) = \left[\left\{ \int_{N-\Delta_L}^{N} K_L (x-N)^2 f_X(x)\, dx \right\} \frac{1}{1-P_r} + \left\{ \int_{N}^{N+\Delta_U} K_U (x-N)^2 f_X(x)\, dx \right\} \frac{1}{1-P_r} \right] \quad (3-10)$$

According to Appendix B, the expected quality loss can be written as:

$$E(QL_{IIR}) = \frac{P_{aL}}{1-P_r} K_L \left[\sigma^2_{T_{D_L}} + \left(\mu_{T_{D_L}} - N \right)^2 \right] + \frac{P_{aU}}{1-P_r} K_U \left[\sigma^2_{T_{D_U}} + \left(\mu_{T_{D_U}} - N \right)^2 \right] \quad (3-11)$$

The total cost for producing one unit of each part for IIR can be expressed below:

$$\begin{aligned} C_{IIR} &= \text{Conversion cost} + \text{Expected Quality Loss} + \text{Expected Inspection Cost} \\ &\quad + \text{Expected Scrap Cost} + \text{Expected Reworking Cost} \\ &= C_C + E(QL_{IIR}) + E(C_{I_{IIR}}) + E(C_{S_{IIR}}) + E(C_{R_{IIR}}) \end{aligned}$$

Approaches for Developing Constraints

The purpose of this research is to develop a model for optimizing tolerance allocation subject to:

(1) a constraint associated with the process variances of the parts to be assembled into the envelope, the optimum process variance of which is specified, in order to satisfy the specified allowable maximum gap standard deviation

(2) a constraint associated with the allowable range for the semi-tolerance zone for each side of each part

(3) a constraint associated with the minimum process capability index required for each side of each part

(4) a constraint associated with the allowable range for the process standard deviation for each part

(5) a constraint associated with the minimum proportion of conforming gaps

The specified minimum semi-tolerance zone for each side of each part is the allowable minimum value for the variation from the nominal size. It is the maximum value chosen from all of the relevant specified values; for example, the minimum value of the semi-tolerance zone is specified by the capability of the machine used and/or by the maximum conversion cost accepted by the manufacturer.

The minimum requirement for the process capability index for each side of each part is the requirement for the semi-tolerance zone having the process capability index not less than the specified minimum value. This also is the maximum value chosen from all of the relevant specified values.

The minimum requirements for quality for many parts are specified in terms of the proportions of conforming parts instead of the process capability indices. Those specified minimum proportions of conforming parts should be the maximum values chosen from all of the relevant specified values; for example, the values are specified by the customers, the quality level goal of the manufacturer, and/or the maximum conversion cost accepted by the manufacturer. Those products can use model OTA with a constraint associated with the minimum requirement for the process capability index, rather than the specified minimum proportion of conformity. This is applicable because the constraint is measured as the number of the standard deviations in the semi-tolerance zone. The detail for transforming the minimum proportion conforming to the minimum requirement for the process capability index is discussed in the section for developing constraints associated with the minimum specified process capability indices.

In order to make model OTA applicable to any product using any one of the inspection strategies for each independent part where necessary, the type of inspection strategy has to be included in the model.

A Constraint Associated with The Allowable Maximum Gap Standard Deviation

The gap standard deviation resulting from the square root of the summation of the variances of the parts and the envelope is an important concern for optimizing tolerance allocation. For instance, the optimum semi-tolerance zones for the parts are set at the maximum values whenever the saving rate due to increases in semi-tolerance zones is greater than the increasing rate of the quality loss. In order to satisfy the ability for assembling every part into the envelope, the square root of the summation of the parts and the envelope variances should be equal to or less than the allowable maximum value of the gap standard deviation. The following relation is used to calculate the standard deviation resulting from the square root of the summation of independent random variables applied in developing this constraint:

If $X_1, X_2, \ldots, X_n$ are n independent random variables with variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$, then the standard deviation of $Y = \sum_{i=1}^{n} a_i X_i$, where $a_1, a_2, \ldots, a_n$ are real constants, is

$$\sigma_Y = \sqrt{\sum_{i=1}^{n} a_i^2 \sigma_i^2} .$$

This research deals with linear tolerance allocation and gives equal priorities to the standard deviations of the dimensions of the gap, the envelope and all of the parts. The constraint associated with the allowable maximum gap standard deviation is written as:

$$\sigma_{\max_G} \geq \sqrt{\sum_{i=1}^{m} \sigma_i^2 + \sigma_E^2} \quad \text{for } i = 1, 2, \ldots, m$$

where

$\sigma_{\max_G}$ = the allowable maximum standard deviation of the gap, and

σ_E^2 = variance of the envelope

The purpose of this constraint is to restrict the semi-tolerance zones in order to meet the finished product quality criterion that is measured in terms of the standard deviation. The allowable maximum standard deviation of the gap of a product can be obtained from many sources: (1) quality standard, (2) customer requirement, (3) value determined from the parameter design, (4) the maximum standard deviation for the product for the next process, and/or (5) available values due to the capabilities of the machines and the tools for the assembly.

Constraints Associated with Allowable Ranges For Semi-Tolerance Zones

The allowable range for the semi-tolerance zone is composed of the minimum and the maximum values allowed for the semi-tolerance zone. The specified minimum semi-tolerance zone for each side of each part is the allowable minimum value for the variation from the nominal size. It must satisfy the minimum values for all of the requirements and conditions that could be specified from: (1) the quality standard, (2) the customers' requirements, (3) the value determined from parameter design, (4) the minimum semi-tolerance zone for the product for the next process, (5) the available values due to the capabilities of the machine(s) and the tool(s) used and/or (6) the maximum conversion cost accepted by the manufacturer. Therefore, it must be the maximum value from those specified values. The allowable minimum semi-tolerance zones are:

$\Delta_L \geq \Delta_{min_L}$ and

$\Delta_U \geq \Delta_{min_U}$.

Where Δ_{min_L} and Δ_{min_U} are the maximum values compared with the specified values from other sources; as a result, they are the allowable minimum values for semi-tolerance zones for model OTA.

Since the allowable maximum semi-tolerance zone must satisfy the maximum values for all of the specified requirements and conditions, it must be the minimum value of all of the maxima. The allowable maximum semi-tolerance zones for each part are written as:

$\Delta_L \leq \Delta_{\max L}$ and

$\Delta_U \leq \Delta_{\max U}$.

Where $\Delta_{\max_L}$ and Δ_{max_U} are the minimum values compared with the specified values from other sources; as a result, they are the allowable maximum values for semi-tolerance zones for model OTA.

Constraints Associated with The Minimum Requirements for Process Capability Indices

The process capability index is sometimes required by the customers. As a result, it has to be included in the model in order to make the tolerance satisfy this requirement. The process capability index is used to measure the ability of the process to manufacture a product that meets the specification limits. There are many types of process capability indices for various purposes of measurements based on specified assumptions and conditions. The model OTA assumes that every process and input for manufacturing a product satisfy all the assumptions required for using the specified minimum process capability index. Those required assumptions (see Pignatiello and Ramberg (1993)) are listed below:

(1) The process is in a state of statistical control without any special cause, significant process drift or process oscillation.

(2) The process capability is determined from the random samples that correctly represent the capability of the population.

(3) Drawing the statistical inferences and constructing the confidence intervals for the process are performed from the data with a normal distribution. For a non-normal distribution, the bootstrapping method (sampling with a large sample size is simulated drawn from an actual sample with a small sample size) or an

approach for transforming a non-normal to a normal distribution needs to be applied.

(4) The consecutive observations are independent of one another.

Applying an appropriate process capability index can give more correct information with more detail about the ability of the process. For instance, measuring the capability of the process with mean offset from nominal by using C_{pm} is more accurate than using C_p because C_{pm} considers the amount of offset in measuring the capability of the process whereas C_p does not. Therefore, choosing an appropriate process capability index is important. We start by considering $C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - N)^2}}$ developed by Chan et al. (1988) The purpose of model OTA is to be able to optimize semi-tolerance zones for parts with means that may be offset from the nominal sizes. As a result, C_{pm} needs to be modified in order to suit the model.

Some process capability indices independently measure the capability of the process manufacturing a product that meets (1) the lower specification limit and the process mean and (2) the process mean and the upper specification limit. The concept of those is not useful for this research because the purpose of the constraint in model OTA is to measure the ability of the process that meets the specification limits and the nominal size. Therefore, this research needs to modify C_{pm} in order to make it be able to measure the ability of the process manufacturing a product that meets (1) the lower specification limit and the nominal size and (2) the nominal size and the upper specification limit. Since the length of the specification limits for C_{pm} is going to be divided into two sections for the process capability indices for the lower and the upper sides, the denominator of the modified indices should be 3 instead of 6. The modified process capability index is:

$$C_{pmL} = \frac{N - LSL}{3\sqrt{\sigma^2 + (\mu - N)^2}} \quad \_\_\_(3-12)$$

$$C_{pmU} = \frac{USL - N}{3\sqrt{\sigma^2 + (\mu - N)^2}} \quad \_\_\_(3-13)$$

where

C_{pmL} = process capability index for the lower side of each part

C_{pmU} = process capability index for the upper side of each part.

The above process capability indices can indicate, over a period of time of operation as a stable process, how the deviation of the produced dimension compares with the semi-tolerance zones.

In real application, the customer specifies the minimum requirements for the process capability indices as formulated in equation (3-12) and (3-13), C^*_{pmL} and C^*_{pmU}. Therefore, the constraints associated with the minimum requirements from the customer for the process capability indices are:

$$\frac{N - LSL}{3\sqrt{\sigma^2 + (\mu - N)^2}} \geq C^*_{pmL} \quad \_\_\_(3-14)$$

$$\frac{USL - N}{3\sqrt{\sigma^2 + (\mu - N)^2}} \geq C^*_{pmU} \quad \_\_\_(3-15).$$

Constraints Associated with Allowable Ranges for Process Standard Deviations

Assumptions of this research are that (1) the optimum settings of the operation speeds of a machine for various values of the process standard deviation have been studied; (2) the optimum process standard deviation has an increasing linear relationship with the tolerance as described in Appendix C; and (3) the assignable causes of the variation have been removed from the process standard deviation. Therefore, the current process standard deviation is due to only unassignable causes of variation, and its allowable range has been studied. Since the value of the process standard deviation directly affects the expected quality loss of the model, it needs to be considered as one of the constraints.

$$\sigma_{min} \leq \sigma \leq \sigma_{max}$$

where

σ_{min} = the allowable minimum process standard deviation of each part. This value is specified based on the capability of the machine and/or process used to manufacture that part.

σ_{max} = the allowable maximum process standard deviation of each part. It is assumed to depend on the economic condition depending on the capability of the machine and/or the process used to manufacture the part.

Constraints Associated with the Specified Minimum Proportions of Conforming Units of the Product

Another important constraint is the one for making sure that the product resulting from assembling parts into the envelope will have a probability of conforming gaps not less than the specified minimum proportion. The constraint associated with the specified minimum proportion of conforming parts or the specified minimum process capability index for each side of each part itself cannot guarantee that the proportion of conformity of the gap resulting from the assembly will meet the specified minimum value. Therefore, the constraints associated with the specified minimum proportions of the gap's conforming units need to be included.

For some problems, the optimum semi-tolerance zones for the lower sides may differ from those for the upper sides because the quality loss coefficients for the lower sides are different from those for the upper sides and/or the process means of the dimensions offset from the nominal sizes. Therefore, the constraint associated with the specified minimum proportion of the conformity for the gap of the assembled product should be independently considered for the lower and the upper sides.

The minimum proportion conforming for each side of the gap may be specified by the customer, the capability of the machine assembling the product, and/or the quality standard of the product. For some products, the value resulting from the multiplication of the specified minimum proportion conforming for every part and that for the envelope may be considered as the minimum requirement for the proportion of the conformity of the gap of the assembled product if the number of the parts is small, and the values of the minimum proportions conforming of the parts are great as well. Finally, the maximum value from all of the specified values is chosen for this constraint. The model OTA of this research assumes that the specified minimum proportions conforming of the gap of the assembled product are measured as 4 standard deviations in the semi-tolerance zones for the upper and the lower sides. And then, the values of the proportions of conforming for each side of the gap are calculated from a normal distribution with the mean equal to the envelope process mean minus the parts process means, and the variance equal to the summation of the parts and the envelope variances. This means that they include the effects of mean offset from the nominal size if they are applied. Let:

$P^*_{aL_G}$ = the value of the specified minimum proportion conforming for the lower side of the gap of the product, and

$P^*_{aU_G}$ = the value of the specified minimum proportion conforming for the upper side of the gap of the product.

The next stage of developing this constraint is determining the proportions conforming for the lower and the upper sides of the gap based on the semi-tolerance zones of the parts being determined. This depends on the distributions of the dimensions and the truncation being discussed in detail in the following sections.

For Normal distributions without Truncation

If the parts and the envelope are not inspected, their dimensions' distributions are not truncated. The dimension of the gap has a normal distribution because every part and the envelope have normal distributions. The mean and the standard deviation of the gap resulting from assembling every part into the envelope are

$$\mu_G = \mu_E - \sum_{i=1}^{m} \mu_i$$

$$\sigma_G = \sqrt{\sigma_E^2 + \sum_{i=1}^{m} \sigma_i^2}\ .$$

The proportions conforming for the below and the above the nominal size of the gap are denoted as P_{aL_G} and P_{aU_G}, respectively.

<u>For Normal distributions with Truncation(s)</u>

For IWR and IIR, if one or more of the dimension(s) of the parts and/or of the envelope is(are) truncated, there is no nice distribution of the dimension of the gap. Therefore, simulating the proportion of conformity of the gap based on specified conditions of the product must be applied to this constraint. The rest of this section shows the approach used in this research for simulating the proportion of conformity of the gap for truncated normal distribution(s).

The number of random numbers being generated for the dimension of each part and for the envelope should be sufficient for simulating their distributions. This means the number of the generated units for each part is sufficiently large to estimate the integer part of the proportion nonconforming specification (such as 3.4 PPM). The proportions of the conforming units based on the specification limits being optimized for the parts, envelope and gap along with time needed for simulating data are important criteria for selecting the appropriate number of random numbers generated. For each of the examples in this research, one million values for the gap from assembling the parts into the envelope will be used.

Due to the memory-intensive nature of this process, one value for the dimension of each part and that of the envelope are generated at a time. The criteria used for rejecting the nonconforming unit for the parts and the envelope for each generation approximate actual practice. This simplifies the simulation program, which can approximate the proportions of the dimension failing (1) outside the lower limit, (2) between the lower limit and the nominal size, (3) between the nominal size and the upper limit, and (4) outside the upper limit. The criteria for rejecting nonconforming units used in this model is explained below. In addition, the practical criteria for rejecting nonconforming units are described in Appendix D.

If at least one of the generated dimensions for the parts and/or that for the envelope is (are) less or greater than the lower or the upper specification limit(s), all generated dimensions for every part and that for the envelope from that iteration of the simulation are rejected. For the case with the generated dimension for every part and that for the envelope conform to their specification limits, the generated dimension for every part is subtracted from the generated dimension for the envelope, then the dimensional result, the gap, is compared to the specification limits of the gap for determining the conformity.

The purpose of the constraints associated with the specified minimum proportions conforming for the product is to control the semi-tolerance zones of the parts to satisfy the specified minimum value for the lower and the upper sides of the gap. Therefore, no matter the inspection strategy (IWR or IIR) selected for the gap, each unit of the assembled product can be categorized into one of three types: (1) a scrap due to its dimension less than the lower specification limit or greater than the upper limit, (2) a conforming unit whose dimension falls between the lower specification limit and the nominal size, or (3) a conforming unit whose dimension falls between the nominal size and the upper specification limit. Then, the dimensions for the next iteration of simulating are generated and compared with the specification limits. The procedure starting from generating dimensions to categorizing the dimension of the gap is repeated until the millionth units of the assembled product counted. Finally, the proportions of (1) reject, (2) conforming units on the left side of the nominal size, and (3) conforming units on the right side of the nominal size are calculated and used in the constraint as shown below:

Let $\breve{X}_{ij}$ = dimension of i[th] part from j[th] generation

$\breve{X}_{Ej}$ = dimension of the envelope from j[th] generation

If $\{(\breve{X}_{Ej} < N_E - \Delta_{LE})$ or $(\breve{X}_{Ej} > N_E + \Delta_{UE})\}$

or $\left[\{(\breve{X}_{ij} < N_i - \Delta_{Li})\right.$ or $\left.(\breve{X}_{ij} > N_i + \Delta_{Ui})\}\text{ for } 1,\ldots,m\right]$

then, reject $\breve{X}_{Ej}$ and $\breve{X}_{ij}$ for $i = 1, 2, \ldots, m$, and for that value of j,

else, classify the gap resulting from subtracting the dimension of the envelope by those of the parts:

Reject where $\left\{(\breve{X}_{Ej} - \sum_{i=1}^{m} \breve{X}_{ij}) < (N_G - \Delta_{LG})\right\}$ or $\left\{(\breve{X}_{Ej} - \sum_{i=1}^{m} \breve{X}_{ij}) > (N_G + \Delta_{UG})\right\}$

Conformity for the lower side where $(N_G - \Delta_{LG}) \le (\breve{X}_{Ej} - \sum_{i=1}^{m} \breve{X}_{ij}) \le N_G$

Conformity for the upper side where $N_G \le (\breve{X}_{Ej} - \sum_{i=1}^{m} \breve{X}_{ij}) \le (N_G + \Delta_{UG})$

Finally, calculate the proportion for each category of the gap

P_{RF} = Proportion of Reject

$$= \frac{\#\text{ of Reject}}{\#\text{ of Reject} + \#\text{ of Conformity for the lower side} + \#\text{ of Conformity for the upper side}}$$

P_{LF} = Proportion of Conformity for the lower side

$$= \frac{\#\text{ of Conformity for the lower side}}{\#\text{ of Reject} + \#\text{ of Conformity for the lower side} + \#\text{ of Conformity for the upper side}}$$

P_{UF} = Proportion of Conformity for the upper side

$$= \frac{\text{\# of Conformity for the upper side}}{\text{\# of Reject + \# of Conformity for the lower side + \# of Conformity for the upper side}}$$

where

\# of reject = the number of the reject from the finished product due to the gap outside the specification limits

\# of conformity for the lower side = the number of the conforming finished product whose gap's dimension falling between the lower specification limit and the nominal size

\# of conformity for the upper side = the number of the conforming finished product whose gap's dimension falling between the nominal size and the upper specification limit

For this research, # of reject + # conforming for the lower side + # conforming for the upper side equals one million.

Finally, the constraints associated with the specified minimum proportions conforming for the lower and the upper sides of the gap resulting from assembling every part into the envelope can be written below:

$$\left\{P_{aL_G}\right\}^{I_{NI_E} * \prod_{i=1}^{m} I_{NI_i}} * \left\{P_{LF}\right\}^{1-(I_{NI_E} * \prod_{i=1}^{m} I_{NI_i})} \geq P^{*}_{aL_G}$$

$$\left\{P_{aU_G}\right\}^{I_{NI_E} * \prod_{i=1}^{m} I_{NI_i}} * \left\{P_{UF}\right\}^{1-(I_{NI_E} * \prod_{i=1}^{m} I_{NI_i})} \geq P^{*}_{aU_G}$$

where

$I_{NI\,E}$ binary indicator that equals 1 for applying non-inspection strategy to the envelope, or equals 0 for not applying non-inspection strategy to the envelope

$I_{NI\,i}$ = binary indicator that equals 1 for applying non-inspection strategy to each part, or equals 0 for not applying non-inspection strategy to each part

The Entire Model OTA

This final section shows the entire model OTA. The model OTA can be applied with any one of the inspection strategies for each part. I_{IWR_i}, I_{IIR_i} and C_T used in the objective function of model OTA are defined as:

$I_{IIR\,i}$ = binary indicator that equals 1 for applying 100% inspection with imperfect reworking strategy to each part, or equals 0 for not applying 100% inspection with imperfect reworking strategy to each part

$I_{IWR\,i}$ = binary indicator that equals 1 for applying 100% inspection without reworking strategy to each part, or equals 0 for not applying 100% inspection without reworking strategy to each part

C_T = grand total cost for manufacturing one unit of the finished product

Model OTA for Optimizing Tolerance Allocation

Objective Function :

$$\text{Minimizing } C_T = \sum_{i=1}^{m}\{I_{NI\,i} * C_{NI\,i}\} + \sum_{i=1}^{m}\{I_{IWR\,i} * C_{IWR\,i}\} + \sum_{i=1}^{m}\{I_{IIR\,i} * C_{IIR\,i}\}$$

where

C_{NI_i} = total cost for producing one unit of each part for NI

= Conversion cost+ Expected Quality Loss

$= C_{C_i} + E\left(QL_{NI_i}\right).$

C_{IWR_i} = total cost for producing one unit of each part for IWR

$= \text{Conversion cost} + \text{Expected Quality Loss}$

$+ \text{Expected Inspection Cost} + \text{Expected Scrap Cost}$

$= C_{C_i} + E\left(QL_{IWR_i}\right) + E\left(C_{I_{IWR_i}}\right) + E\left(C_{S_{IWR_i}}\right).$

C_{IIR_i} = total cost for producing one unit of each part for IIR

$= \text{Conversion cost} + \text{Expected Quality Loss} + \text{Expected Inspection Cost}$

$+ \text{Expected Scrap Cost} + \text{Expected Reworking Cost}$

$= C_{C_i} + E\left(QL_{IIR_i}\right) + E\left(C_{I_{IIR_i}}\right) + E\left(C_{S_{IIR_i}}\right) + E\left(C_{R_{IIR_i}}\right)$

Subject to:

1. A constraint associated with the allowable maximum gap standard deviation:

$$\sigma_G \geq \sqrt{\sum_{i=1}^{m} \sigma_i^2 + \sigma_E^2}$$

2. Constraints associated with allowable ranges for semi-tolerance zones:

$$\Delta_{\min L_i} \leq \Delta_{L_i} \leq \Delta_{\max L_i} \quad \text{for } i = 1, 2, ..., m, \quad \text{and}$$
$$\Delta_{\min U_i} \leq \Delta_{U_i} \leq \Delta_{\max U_i} \quad \text{for } i = 1, 2, ..., m$$

3. Constraints associated with the minimum requirements for the process capability indices:

$$\frac{N_i - LSL_i}{3 * \sqrt{\sigma_i^2 + (\mu_i - N_i)^2}} \geq C^*_{pmL_i} \quad \text{for } i = 1, 2, ..., m, \quad \text{and}$$
$$\frac{USL_i - N_i}{3 * \sqrt{\sigma_i^2 + (\mu_i - N_i)^2}} \geq C^*_{pmU_i} \quad \text{for } i = 1, 2, ..., m$$

4. Constraints associated with allowable ranges for process standard deviations of the parts:

$$\sigma_{\min_i} \leq \sigma_i \leq \sigma_{\max_i} \quad \text{for } i = 1, 2, ..., m$$

5. Constraints associated with the specified minimum proportions conforming of the product

$$\left\{P_{aL_G}\right\}^{I_{NI_E} \cdot \prod_{i=1}^{m} I_{NI_i}} * \left\{P_{L_F}\right\}^{1-(I_{NI_E} \cdot \prod_{i=1}^{m} I_{NI_i})} \geq P^*_{aL_G}$$

$$\left\{P_{aU_G}\right\}^{I_{NI_E} \cdot \prod_{i=1}^{m} I_{NI_i}} * \left\{P_{U_F}\right\}^{1-(I_{NI_E} \cdot \prod_{i=1}^{m} I_{NI_i})} \geq P^*_{aU_G}$$

Table 3-1. Notation Definitions

Notations	Definitions
A	fixed conversion cost for producing one unit of each part
B	conversion cost coefficient due to linear function of tolerance for producing one unit of each part
C_A	raw conversion cost, measured as the unit of user's currency that is US$ for numerical examples in this research, for producing one unit of each part
C_C	total conversion cost for producing one unit of each part.
C_{Dong}	production cost-tolerance function proposed by Dong et al. that can be chosen from 11 cost functions for the same process used for manufacturing the part
C^{*}_{Dong}	the most appropriate production cost-tolerance function chosen from one of the first four best-fit costs with uncomplicated function. It is the most appropriate-fit to the conversion cost-tolerance curve for producing one unit of each part
C	conversion cost coefficient due to quadratic function of tolerance for producing one unit of each part
$C_{I_{NI}}$	inspection cost for one inspected unit of each part for NI
$C_{I_{IWR}}$	inspection cost for one inspected unit of each part for IWR
$C_{I_{IIR}}$	inspection cost for one inspected unit of each part for IIR
C_{IIR}	total cost for producing one unit of each part for 100% inspection with imperfect reworking strategy

Table 3-1. Notation Definitions (Continued)

Notations	Definitions
C_{IWR}	total cost for producing one unit of each part for 100% inspection without reworking strategy
C_M	multiplier of conversion cost for calculating the raw conversion cost for producing one unit of each part. It is similar to the unit cost of casting process that is used as the reference for determining the relative cost for other processes in Dong et al's cost model
C_{NI}	total cost for producing one unit of each part for non-inspection strategy
C_{pm}	process capability index for each part
C_{pmL}	index for measuring the capability of the process producing the dimension below the nominal size
C_{pmU}	index for measuring the capability of the process producing the dimension upper the nominal size
C^{*}_{pmL}	the specified minimum value for C_{pmL}
C^{*}_{pmU}	the specified minimum value for C_{pmU}
$C_{R_{IIR}}$	reworking cost for a reworked unit of each part for IIR
$C_{S_{IWR}}$	scrap cost for a scrapped unit of each part for IWR
$C_{S_{IIR}}$	scrap cost for a scrapped unit of each part for IIR
C_T	grand total cost for manufacturing one unit of the finished product

Table 3-1. Notation Definitions (Continued)

Notations	Definitions
D	conversion cost coefficient due to cubic polynomial of tolerance for producing one unit of each part
$E(\bullet) = \mu_\bullet$	expected value of its argument
F	conversion cost coefficient due to fourth-order polynomial of tolerance for producing one unit of each part
F_I	inspection cost that equals F_I percent of the conversion cost for each part
F_S	scrap cost that equals F_S percent of the conversion cost for each part
F_R	reworking cost that equals F_R percent of the conversion cost for each part
$f_X(x)$	the probability density function for the dimension of the manufactured units of each part
	note font change below
I_{IIR}	binary indicator that equals 1 for applying 100% inspection with imperfect reworking strategy to each part, or equals 0 for not applying 100% inspection with imperfect reworking strategy to each part
I_{IWR}	binary indicator that equals 1 for applying 100% inspection without reworking strategy to each part, or equals 0 for not applying 100% inspection without reworking strategy to each part
$I_{NI\,E}$	binary indicator that equals 1 for applying non-inspection strategy to the envelope, or equals 0 for not applying non-inspection strategy to the envelope

Table 3-1. Notation Definitions (Continued)

Notations	Definitions
I_{NIG}	binary indicator that equals 1 for applying non-inspection strategy to the gap, or equals 0 for not applying non-inspection strategy to the gap
I_{NI}	binary indicator that equals 1 for applying non-inspection strategy to each part, or equals 0 for not applying non-inspection strategy to each part
K	quality loss coefficient for each part (a constant)
K_L	quality loss coefficient for lower side of each part (a constant)
K_U	quality loss coefficient for upper side of each part (a constant)
L_E	the number of standard deviations in the semi-tolerance zone for the envelope, which is not expressed in any unit.
L_G	the number of standard deviations in the semi-tolerance zone for the gap, which is not expressed in any unit
L	the number of standard deviations in the semi-tolerance zone for each part, which is not expressed in any unit
L_{LE}	the specified minimum number of standard deviations in the lower semi-tolerance zone for the envelope, which is not expressed in any unit
L_{LG}	the specified minimum number of standard deviations in the lower semi-tolerance zone for the gap, which is not expressed in any unit
L_L	the specified minimum number of standard deviations in the lower semi-tolerance zone for each part, which is not expressed in any unit
LSL	lower specification limit for each part

Table 3-1. Notation Definitions (Continued)

Notations	Definitions
LSL^*	the maximum value of the lower specification limit in order to satisfy the specified minimum requirement for the process capability index for the lower side that is measured as the number of standard deviations in the semi-tolerance zone. As is result, $N - LSL^* = L_L * \sigma$.
L_{UE}	the specified minimum number of standard deviations in the upper semi-tolerance zone for the envelope, which is not expressed in any unit
L_{UG}	the specified minimum number of standard deviations in the upper semi-tolerance zone for the gap, which is not expressed in any unit
L_U	the specified minimum number of standard deviations in the upper semi-tolerance zone for each part, which is not expressed in any unit
N	nominal size (target value) for each part
N_E	nominal size (target value) for the envelope
N_G	nominal size (target value) for the gap
P_a	$= \int_{N-\Delta_L}^{N+\Delta_U} f_X(x)\,dx$ = proportion of conformity for a new manufactured unit of each part
P_{aL}	$= \int_{N-\Delta_L}^{N} f_X(x)\,dx$ = proportion of conformity below the nominal for a new manufactured unit of each part

Table 3-1. Notation Definitions (Continued)

Notations	Definitions
P_{aL_G}	the proportion of conforming units of the product falling below the nominal (for a case with at least one part and/or the envelope is inspected)
$P^{*}_{aL_G}$	the specified minimum proportion of conforming units of the product falling below the nominal
P_{aU}	$= \int_{N}^{N+\Delta_U} f_X(x)dx$ = proportion of conformity upper the nominal for a new manufactured unit of each part
P_{aU_G}	the proportion of conforming units of the product falling above the nominal (for a case with at least one part and/or the envelope is inspected)
$P^{*}_{aU_G}$	the specified minimum proportion of conforming units of the product falling above the nominal
P_L	$= \int_{-\infty}^{N} f_X(x)dx$
P_r	$= \int_{N+\Delta_U}^{\infty} f_X(x)dx$
P_s	$= \int_{-\infty}^{N-\Delta_L} f_X(x)dx$
P_U	$= \int_{N}^{\infty} f_X(x)dx$

Table 3-1. Notation Definitions (Continued)

Notations	Definitions
P_{LF}	proportion of conforming units of the finished product whose gap falling between the lower specification limit and the nominal size (derived from simulation)
P_{RF}	proportion of reject of the finished product due to its gap outside the specification limits (derived from the simulation)
P_{UF}	proportion of conforming units of the finished product whose gap falling between the nominal size and the upper specification limit (derived from simulation)
T_{min}	the allowable minimum value of the tolerance for each part based on the capability of the machine and/or process used to manufactured the part
T_{max}	the allowable maximum value of the tolerance for each part based on the capability of the machine and/or process used to manufactured the part
USL	the upper specification limit for each part
USL^{*}	the minimum value of the upper specification limit in order to satisfy the specified minimum requirement for the process capability index for the upper side that is measured as the number of standard deviations in the semi-tolerance zone.
$Var(\bullet) = \sigma_{\bullet}^{2}$	= variance of its argument
$\sigma_{\bullet}$	= standard deviation of its argument
X	random dimension produced for each unit for each of *m* part (It is a variable.)

Table 3-1. Notation Definitions (Continued)

Notations	Definitions
X_G	the random dimension for the gap resulting from assembling every part into the envelope. It has mean μ_G and standard deviation σ_G. (for a case with non-inspection for every part and for the envelope)
Δ	tolerance for each part
Δ_{LE}	specified lower semi-tolerance zone for the envelope
Δ_{LG}	specified lower semi-tolerance zone for the gap
Δ_L	lower semi-tolerance zone for each part being optimized
Δ_L^*	$=\Delta_L+(\mu-N)$
Δ_U^*	$=\Delta_U-(\mu-N)$
$\Delta_{\max L}$	the allowable maximum value for the lower semi-tolerance zone for each part (used in constraint type 2)
$\Delta_{Max L}$	the allowable maximum value for the lower semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured the part (It is used to determine the process standard deviation that has an increasing linear relationship with the tolerance)
$\Delta_{\max U}$	the allowable maximum value for the upper semi-tolerance zone for each part (used in constraint type 2)

Table 3-1. Notation Definitions (Continued)

Notations	Definitions
Δ_{Max_U}	the allowable maximum value of the upper semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured the part (It is used to determine the process standard deviation that has an increasing linear relationship with the tolerance)
Δ_{MinA_L}	the allowable minimum value of the lower semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured that part (It is used to determine the process standard deviation that has an increasing linear relationship with the tolerance)
Δ_{MinA_U}	the allowable minimum value of the upper semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured that part (It is used to determining the process standard deviation that that has an increasing linear relationship with the tolerance)
$\Delta_{\min L}$	the allowable minimum value for the lower semi-tolerance zone for each part
$\Delta_{\min U}$	the allowable minimum value for the upper semi-tolerance zone for each part
$\Delta_{U E}$	specified upper semi-tolerance zone for the envelope
$\Delta_{U G}$	specified upper semi-tolerance zone for the gap
Δ_U	upper semi-tolerance zone for each part being optimized
$\sigma_{\max_G}$	the allowable maximum standard deviation of the gap

Table 3-1. Notation Definitions (Continued)

Notations	Definitions
σ_{max}	the allowable maximum process standard deviation of each part. It is assumed to depend on the economic condition depending on the capability of the machine
σ_{min}	the allowable minimum process standard deviation of each part. Model OTA of this research assumes that this value is specified based on the capability of the machine and/or process used to manufacture that part.
# of conformity for the lower side =	the number of the conforming finished product whose gap's dimension falling between the lower specification limit and the nominal size
# of conformity for the upper side =	the number of the conforming finished product whose gap's dimension falling between the nominal size and the upper specification limit
# of reject =	the number of the reject from the finished product due to the gap outside the specification limits

CHAPTER 4

MODEL VERIFICATIONS AND AN EXAMPLE

Model Verifications

The model for optimizing tolerance allocation proposed in this research requires verifications to ensure that its concepts are correct. For illustration, verifications are shown in three sections: verifying effects of (1) changes in means, (2) quality loss coefficients, and (3) constraints. Each section has its own original condition in order to show verification clearly.

Verifying Effects of Changes in Means

A product as shown in Figure 4-1 consisting of three parts assembled within an envelope is chosen to reallocate semi-tolerance zones for minimizing total cost. The nominal size and the semi-tolerance zones for the envelope are specified at 130.1 and 0.075 mm, respectively; and the specified nominal size and semi-tolerance zones for the gap are 0.17 and 0.16 mm, respectively. In addition, the nominal size for Part 1, 2 and 3 are specified as 50.455, 40.725 and 38.75 mm, respectively. Based on empirical studies, the means of the produced dimensions are 50.459, 40.729, 38.746, 130.106 and 0.172 mm for part 1, part 2, part 3, the envelope and the gap, respectively, and the specified standard deviation is 0.013 mm for the envelope. The allowable maximum standard deviation of the gap is 0.029 mm. The produced dimensions of the parts, envelope and gap are normally distributed. The production cost-tolerance model for the fourth-order polynomial function for face milling developed by Dong et al. is used as the conversion cost for every part in this model. The conversion, inspection, scrap and reworking costs, and the quality loss coefficient of each part have been determined. The inspection cost for each inspected unit, the scrap cost for each scrapped unit and the reworking cost for each reworked unit (measured as percent of the conversion cost) are 10, 200 and 25% for every part, respectively. The multiplier for calculating raw conversion cost for producing one unit of part 1, 2 and 3 are 25, 20 and 19, respectively. The allowable maximum semi-tolerance zones for the upper and the lower sides of every part are 0.085 mm. The desired minimum values for process capability indices measured as the numbers of standard deviations in the semi-tolerance zones are 4.0 for both sides of every part. The product is depicted in Figure 4-2; in addition, the necessary data for verifying the effects of changes in means are shown in Table E-1.

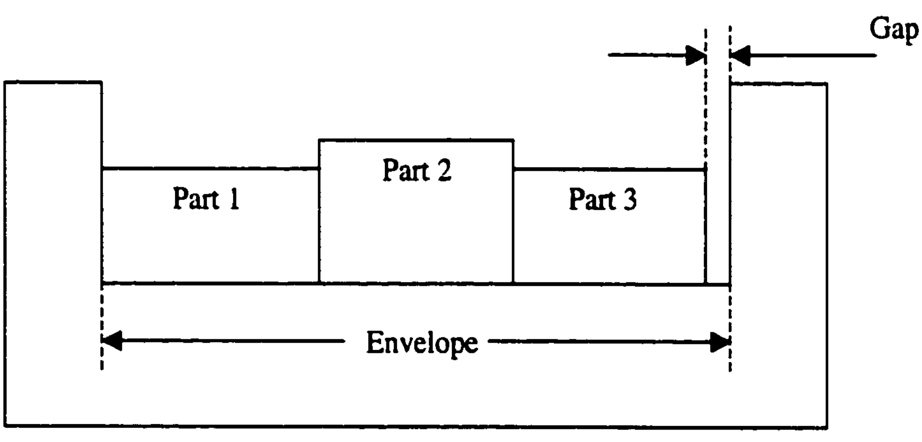


Figure 4-1. A Product Consisting of 3 Parts Assembled in An Envelope

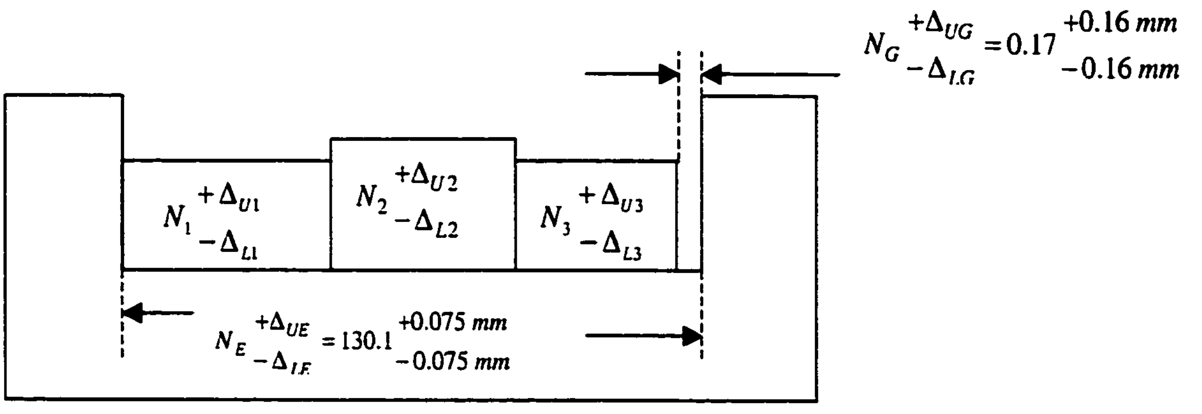


Figure 4-2. A Product for Verifying Effects of Changes in Means

C_{C_i} = conversion cost for producing one unit of ith part for any one of the inspection strategy

$$= \left\{ \begin{array}{l} \left[\left\{ \begin{array}{l} 280.7 - 2407\{2\Delta^{*}_{L_i}\} + 282.3\{2\Delta^{*}_{L_i}\}^2 \\ + 45960\{2\Delta^{*}_{L_i}\}^3 - 106100\{2\Delta^{*}_{L_i}\}^4 \end{array} \right\} \left\{ \frac{P_{aL_i}}{P_a} \right\} \left\{ \frac{1}{100} \right\} + 1 \right] C_{M\,i} \\ + \left[\left\{ \begin{array}{l} 280.7 - 2407\{2\Delta^{*}_{U_i}\} + 282.3\{2\Delta^{*}_{U_i}\}^2 \\ + 45960\{2\Delta^{*}_{U_i}\}^3 - 106100\{2\Delta^{*}_{U_i}\}^4 \end{array} \right\} \left\{ \frac{P_{aU_i}}{P_a} \right\} \left\{ \frac{1}{100} \right\} + 1 \right] C_{M\,i} \end{array} \right\}$$

C_{NI_i} = total cost for producing one unit of ith part for non – inspection strategy

= Conversion Cost + Expected Quality Loss

$$= C_{C_i} + E(QL_{NI_i})$$

C_{IWR_i} = total cost for producing one unit of ith part for 100% inspection without reworking strategy

= Conversion cost + Expected Quality Loss + Expected Inspection Cost + Expected Scrap Cost

$$= C_{C_i} + E(QL_{IWR_i}) + E(C_{I_{IWR_i}}) + E(C_{S_{IWR_i}})$$

C_{IIR_i} = total cost for producing one unit of i[th] part for 100% inspection with imperfect reworking strategy

= Conversion Cost + Expected Quality Loss + Expected Inspection Cost + Expected Scrap Cost + Expected Reworking Cost

$$= C_{C_i} + E\left(QL_{IIR_i}\right) + E\left(C_{I_{IIR_i}}\right) + E\left(C_{S_{IIR_i}}\right) + E\left(C_{R_{IIR_i}}\right)$$

The model of this example is expressed below:

Objective Function :

$$\text{Minimizing } C_T = \sum_{i=1}^{3}\{I_{NIi} * C_{NIi}\} + \sum_{i=1}^{3}\{I_{IWRi} * C_{IWRi}\} + \sum_{i=1}^{3}\{I_{IIRi} * C_{IIRi}\}$$
$$= C_{NI3} + C_{IWR2} + C_{IIR1}$$

Subject to :

1. A constraint associated with the allowable maximum gap standard deviation:

$$\sigma_G \geq \sqrt{\sum_{i=1}^{3}\sigma_i^2 + \sigma_E^2} \quad \text{that is}$$

$$0.029 \geq \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 0.013^2} \qquad \text{(Constraint 11)}$$

2. Constraints associated with allowable ranges for semi-tolerance zones

$\Delta_{minLi} \leq \Delta_{Li} \leq \Delta_{maxLi}$ for i=1, 2 and 3 that are

$0.055 \leq \Delta_{L1} \leq 0.085$ (Constraint 21)

$0.055 \leq \Delta_{L2} \leq 0.085$ (Constraint 22)

$0.055 \leq \Delta_{L3} \leq 0.085$ (Constraint 23)

and

$\Delta_{minUi} \leq \Delta_{Ui} \leq \Delta_{maxUi}$ for i=1, 2 and 3 that are

$0.055 \leq \Delta_{U1} \leq 0.085$ (Constraint 24)

$0.055 \leq \Delta_{U2} \leq 0.085$ (Constraint 25)

$0.055 \leq \Delta_{U3} \leq 0.085$ (Constraint 26)

3. Constraints associated with the minimum requirements for process capability indices:

$$N_i - LSL_i \geq C^*_{pmL_i} * 3 * \sqrt{\sigma_i^2 + (\mu_i - N_i)^2} \quad \text{for } i = 1, 2 \text{ and } 3$$

that are

$$50.455 - LSL_1 \geq C^*_{pmL_1} * 3 * \sqrt{\sigma_1^2 + (50.459 - 50.455)^2} \qquad \text{(Constraint 31)}$$

$$40.725 - LSL_2 \geq C^*_{pmL_2} * 3 * \sqrt{\sigma_2^2 + (40.729 - 40.725)^2} \qquad \text{(Constraint 32)}$$

$$38.75 - LSL_3 \geq C^*_{pmL_3} * 3 * \sqrt{\sigma_3^2 + (38.746 - 38.75)^2} \qquad \text{(Constraint 33)}$$

and

$$USL_i - N_i \geq C^*_{pmU_i} * 3 * \sqrt{\sigma_i^2 + (\mu_i - N_i)^2} \quad \text{for } i = 1, 2 \text{ and } 3$$

that are

$$USL_1 - 50.455 \geq C^*_{pmU_1} * 3 * \sqrt{\sigma_1^2 + (50.459 - 50.455)^2} \qquad \text{(Constraint 34)}$$

$$USL_2 - 40.725 \geq C^*_{pmU_2} * 3 * \sqrt{\sigma_2^2 + (40.729 - 40.725)^2} \qquad \text{(Constraint 35)}$$

$$USL_3 - 38.75 \geq C^*_{pmU_3} * 3 * \sqrt{\sigma_3^2 + (38.746 - 38.75)^2} \qquad \text{(Constraint 36)}$$

4. Constraints associated with allowable ranges for process standard deviations of the parts

$\sigma_{\min i} \leq \sigma_i \leq \sigma_{\max i}$ for i = 1, 2 and 3 that are

$$0.012 \leq \sigma_1 \leq 0.0156 \qquad \text{(Constraint 41)}$$

$$0.012 \leq \sigma_2 \leq 0.0156 \qquad \text{(Constraint 42)}$$

$$0.012 \leq \sigma_3 \leq 0.0156 \qquad \text{(Constraint 43)}$$

5. Constraints associated with the specified minimum proportions of conforming units of the product

$$\{P_{aL_G}\}^{I_{NI_E} \cdot \prod_{i=1}^{3} I_{NI_i}} * \{P_{LF}\}^{1-(I_{NI_E} \cdot \prod_{i=1}^{3} I_{NI_i})} \geq P^{*}_{aL_G} \quad \text{that is}$$

$$P_{L_F} \geq P^{*}_{aL_G} \qquad \text{(Constraint 51)}$$

and

$$\{P_{aU_G}\}^{I_{NI_E} \cdot \prod_{i=1}^{3} I_{NI_i}} * \{P_{UF}\}^{1-(I_{NI_E} \cdot \prod_{i=1}^{3} I_{NI_i})} \geq P^{*}_{aU_G} \quad \text{that is}$$

$$P_{U_F} \geq P^{*}_{aU_G} \qquad \text{(Constraint 54)}$$

Results from verifying the effects of changes in means expressed in table E-2 are described as follows.

Conversion costs are about 82, 83 and 88 percent of the total costs whereas expected quality losses were about 10, 9, and 8 to 13 percents for part 1, 2 and 3, respectively. Based on the concept applied to calculating the conversion costs for the lower and the upper sides of each part, the semi-tolerance zone for the side containing the process mean has a higher proportion of conforming units than that for the other side although that part has the same lengths of the semi-tolerance zones for both sides. Based on comparing two parts having the same lengths of tolerances, the part with the semi-tolerance zone for the side containing the process mean longer than that for the other side has lower expected total cost than the part with the shorter semi-tolerance zone containing the process mean. Therefore, the model proposed in this research should give the semi-tolerance zones for the sides containing the process means longer than those for the other sides, and the results of verifying the model are as expected.

The optimum semi-tolerance zones for the upper and the lower sides are changed from unequal to equal when the process mean offset(s) from the nominal size(s) is (are) eliminated. When the same amount of the process mean offset from the nominal switches to the opposite side, the values of the optimum semi-tolerance zones for the lower and the upper sides switch to each other as well

Since the original condition for verifying the effects of changes in means have conversion costs more than 80 percent of the total costs for all parts, the model tries to set the values of the semi-tolerance zones as great as possible in order to decrease the total costs. Nevertheless, the constraint associated with the gap standard deviation resulting from the square root of the summation of the parts and the envelope variances is always binding. For cases with the process means of part 1 not equal to the nominal size, five out of six of the optimum semi-tolerance zones are less than the allowable maximum values because of the limitation of the allowable maximum process standard deviation of the gap. Therefore, the optimum semi-tolerance zone on the side containing the process mean of part 1 is the greatest and equal to the allowable maximum value due to part 1 having the highest multiplier for calculating the raw conversion cost. Since the multiplier for the conversion cost for part 2 is greater than that for part 3, the optimum semi-tolerance zone on the side containing the process mean for part 2 is longer than that for part 3 as expected. The optimum semi-tolerance zones for the sides not containing the process means ranked from the greater to the smaller are the semi- tolerance zones of part 1, 2 and 3, respectively.

Generally, for a condition with (1) the conversion costs far higher than the expected quality losses, and (2) the greatest conversion cost far higher than the rest, the constraint associated with the minimum requirement for process capability index for the part with the very small conversion cost(s) is (are) likely to be binding. As a result, the constraint associated with the minimum requirement for process capability index for the lower or the upper side for part 3 is binding for some cases, whereas those for part 1 and part 2 are not, as expected.

For the condition with the process mean of every part equal to the nominal size, the optimum semi-tolerance zone for the lower side is equal to that for the upper side of each part although the quality loss coefficients for the lower sides of part 1 and 2 are 150 percent of those for the upper sides, and vice versa for part 3. This results from the proportions of the conversion costs being very high in percentages of the total costs compared with those of the expected quality losses. Therefore, the differences in quality loss coefficients cannot make the values of semi-tolerance zones for the lower sides different from those for the upper sides.

The results from verifying the effects of changes in means demonstrate that the model proposed in this research responds appropriately and as expected to changes in model parameters. The next section deals with verifying the effects of quality loss coefficients. For clearly demonstrating the effects of the quality loss coefficients, the original condition should be changed. For instance, the new original condition has the process mean equal to the nominal size for each part, and the quality loss coefficient for the lower side equal to that for the upper side for every part. The original condition is shown in table E-3.

Verifying Effects of Quality Loss Coefficients

Results from verifying the effects of quality loss coefficients shown in table E-4 are described as follows.

For a part with non-inspection strategy, the values of the semi-tolerance zones for both sides do not affect the expected quality losses whereas the values of quality loss coefficients affect the expected quality losses. The constraint associated with the allowable maximum process standard deviation of the gap, constraint 11, is binding for the chosen original condition, and it is always binding for all conditions changed. For a condition with the quality loss coefficient for the lower side, the upper side, or both the lower and the upper sides being increased, the semi-tolerance zones should be changed in order to optimize the new solution. In order to minimize the total conversion cost for part 1 with non-inspection strategy, its upper and lower semi-tolerance zones are decreased with the same amount although its quality loss coefficient for only one side is increased.

The optimum semi-tolerance zones for both sides of part 2 with 100% inspection without reworking strategy for the original condition are equal. Nevertheless, the optimum semi-tolerance zone for the lower side of part 3 with 100% inspection with imperfect reworking strategy is greater than that of the upper side. This results from the scrap cost being greater than the reworking cost. Increasing the quality loss coefficient for either side of part 2 or part 3 makes the semi-tolerance zone for that side decrease. However, the tolerance resulting from the summation of the semi-tolerance zones for both sides of all three parts remains the same for every case. Therefore, decreasing the semi-tolerance zone for the side of a part makes the value(s) of other semi-tolerance zone(s) increase. The semi-tolerance zone with the greatest expected conversion cost for the parts without changing in quality loss coefficient is the first one to increase in the semi-tolerance zone, and by the greatest amount. Later, the semi-tolerance zone with the second greatest expected conversion cost will increase, and then the ones in the next consecutive ranks unless constraint 11 is binding. The decreased amount of the semi-tolerance zone for a side of part 2 is equal to the decreased amount of that for the other side when the same amounts of increasing the quality loss coefficients has been switched from one side to the other side. This results from the nonconforming unit being scrapped whether it is greater than the upper limit or it is smaller than the lower limit.

For a combination of simultaneously increasing the quality loss coefficients for 2 parts, the optimum semi-tolerance zones for the sides with increasing quality loss coefficients are decreased. For increasing the quality loss coefficient for every part, the semi-tolerance zone for the side with increase in the quality loss coefficient for part 2 is decreased by the biggest amount. This results in increasing the semi-tolerance zones for part 1.

The results from verifying the effects of quality loss coefficients show that the proposed model of this research can be accepted, and continues to the last section for verifying the effects of changing in constraints.

Verifying Effects of Constraints

Verifying the effects of constraints is the last section for verifying the proposed model of this research. Only one constraint is changed at a time in order to clearly verify the model. Another original condition is needed and shown in table E-5. The results from verifying the effects of changing constraints shown in table E-6 are expressed as follows:

Changing in Allowable Maximum Process Standard Deviation of The Gap

The optimal solution for the original condition shows that (1) the semi-tolerance zones for the upper sides of part 1 and part 2, and that for the lower side of part 3 are set at the allowable maximum values, and (2) constraints 11, 23, 24 and 25 are binding. The tolerance resulting from the summation of the semi-tolerance zones for every part is positively related to the allowable maximum process standard deviation of the gap. As a result, the optimum semi-tolerance zones smaller than the allowable maximum values are increased when the allowable maximum process standard deviation of the gap is increased. In contrast, all of the optimum semi-tolerance zones except that for the upper side of part 1 are decreased when the allowable maximum process standard deviation of the gap is decreased. The ranks of the values of the optimum semi-tolerance zones from the greatest to the smallest corresponds to the ranks of the values of the conversion costs from the greatest to the smallest. They are the semi-tolerance zone for the upper side of part 1, the upper of part 2, the lower of part 3, the lower of part 1, the lower of part 2 and the upper of Part 3. As can be seen, the upper semi-tolerance zone for part 1 remains equal to its allowable maximum value.

Changing in Allowable Maximum Values of Semi-Tolerance Zones

The concept of the relationship between the process standard deviation and the tolerance for studying the sensitivity in the total cost to changing in the allowable maximum tolerance is discussed in detail in appendix F. The value of the allowable maximum process standard deviation remains the same while the allowable maximum tolerance is changed. At any values of the process standard deviations, the tolerances for the case with reduction in the process standard deviation are the smallest; those for the original condition are next; and those for the case with increasing the allowable maximum tolerance are the greatest.

From the model verification, increasing the allowable maximum value of the semi-tolerance zone for any side of any part makes the optimum value of that semi-tolerance zone increase except that already set at the allowable maximum value. The result from reduction in the allowable maximum value for the semi-tolerance zone for any side of any part makes the optimum value of that semi-tolerance zone decrease, as it should be. For a case with increasing or decreasing the allowable maximum tolerance, its optimum process standard deviation at a fixed value of the tolerance is changed. The constraint associated with the process standard deviation resulting from the square root of the summation of the parts and the envelope variances is always binding for every case in order to minimize the total cost. Therefore, when the optimum process standard deviation of a part is decreased due to increasing the allowable maximum tolerance, at least one of the semi-tolerance zones for the rest must increase in order to use all of the allowable maximum process standard deviation of the gap. As a result, although the semi-tolerance zone for the side of the part with increase in the allowable maximum tolerance is increased, at least one of the semi-tolerance zones for the rest is increased as well. For instance, increasing the allowable maximum semi-tolerance zone for the lower side of part 1 from 0.085 to 0.09 mm makes (1) the lower semi-tolerance zone for part 1 increase from 0.078 to 0.079 mm, (2) the lower semi-tolerance zone for part 2 increase from 0.073 to 0.075 mm, and (3) the upper semi-tolerance zone for part 3 increase from 0.069 to 0.071 mm. The results from verifying the effects of increasing and decreasing the allowable maximum process standard deviation for each side of each part are as they should be. The next step is verifying the effects of changing in constraints associated with the minimum requirements for process capability indices.

Changing in Minimum Requirements for Process Capability Indices

The minimum requirements for process capability indices are measured as the numbers of the standard deviations in the semi-tolerance zones. The minimum requirement for process capability index for each side of each part for the original condition is specified at 4. The optimum solution for the original condition gives the semi-tolerance zones for the upper sides of part 1 and 2, and that for the lower side of part 3 equal to the allowable maximum values. The process capability indices measured as the numbers of standard deviations in the optimum semi-tolerance zones for the lower and the upper sides of part 1, part 2 and part 3 are 5.10, 5.56, 4.81, 5.60, 5.51 and 4.47, respectively. For cases with the new specified minimum requirements for process capability indices for model verification remaining equal to or less than the process capability indices for the optimum solution of the original condition, the optimum values of those semi-tolerance zones for the new conditions are not changed. In contrast, for cases with the new specified minimum requirements for process capability indices greater than the process capability indices for the optimum solution for the original condition, the new optimum values of those semi-tolerance zones are increased to equal to or greater than the new specified minimum requirements. In addition, a case with the new specified minimum requirement for the process capability index greater than the allowable maximum value of that semi-tolerance zone gives an infeasible solution. This can be seen for the case with the minimum requirement for the process capability index for the upper side of part 1 is changed from 4 to 6. Based on the chosen original condition, the model verification gives the results as expected.

Changing in Allowable Maximum Process Standard Deviations of the Parts

The concept of the relationship between the process standard deviation and the tolerance for studying the sensitivity in the total cost to changing in the allowable maximum process standard deviation is described in appendix F. The value of the allowable maximum tolerance remains the same while the value of the allowable maximum process standard deviation is changed. Changing in the allowable maximum process standard deviation, no matter whether increasing or decreasing, makes the value of the optimum process standard deviation at any fixed tolerance change. Based on the chosen original condition, increasing the allowable maximum process standard deviation of a part makes the semi-tolerance zone(s) for that part and at least one of the semi-tolerance zones for other parts reduce in order to satisfy the constraint associated with the allowable maximum process standard deviation of the gap. The allowable maximum process standard deviation of each part is inversely related to its optimum semi-tolerance zones. For the chosen original condition, the constraint associated with the process standard deviation resulting from the square root of the summation of the parts and the envelope variances is always binding for every case of verification in order to minimize the total cost. As a result, decreasing an allowable maximum process standard deviation makes not only the optimum semi-tolerance zone(s) of that part but also at least one of the optimum semi-tolerance zones of the rest increase in order to use all of the allowable maximum process standard deviation of the gap. The results from this verification also support the correctness of the proposed model.

Changing in Specified Minimum Proportions of Conforming Units of the Product

In order to show the effects of the specified minimum proportions of conforming units of the product, another original condition is needed and shown in table E-7. The actual dimension of the gap depends on the produced dimensions of the parts and envelope. In addition, the alternatives for the lengths for the parts and envelope that could be assembled into a specified length of the gap is very large. Since the gap also has allowable ranges in lengths for the lower and the upper sides of its nominal size, there are several combinations with many alternatives among the dimensions of the parts and envelope for producing a conforming unit of the product. The new original condition chosen in this example can verify the proposed model only for a case with increasing the specified minimum proportions of conforming units for both lower and upper sides. The optimum semi-tolerance zones are not changed for cases with the new specified minimum proportions of conforming units of the product not greater than the numbers of standard deviations in the optimum semi-tolerance zones of the gap for the original condition as expected. In addition, the model has no feasible solution for a case with at least one of the specified minimum proportions of conforming units of the product set at the value, such as 5.5 for the upper side, greater than the allowable maximum value, such as 5.036 for this example.

A Numerical Example Including Applying Design of Experiments to Sensitivity Analysis

The rest of this chapter deals with a numerical example for optimizing semi-tolerance zones, and applying designs of experiments to studying the sensitivity in the total cost to the quality loss coefficients and the constraints. Analyzing the sensitivity of the total cost, but not studying cost surfaces, is performed in this research because the grand total cost of the constructed numerical example is not sensitive to the optimum semi-tolerance zones. In addition, it has multiple local optimum semi-tolerance zones due to totally six semi-tolerance zones. There are large numbers of combinations for the semi-tolerance zones for each value of the grand total cost. The author of this research could not analyze the behavior of the cost surfaces from the preliminary study. Finally, DOE is applied to sensitivity analysis.

Screening significant factors by using Plackett-Burman Design cannot be analyzed if any one of the solutions for experimental runs is missing. As a result, another original condition for this numerical example is needed in order to make it have feasible solutions for every condition, although it does not represent the real problem that has infeasible solutions for some conditions. All necessary information of the chosen original condition for the example is shown in table E-9.

Screening for Significant Single Factors

As was discussed in Chapter 3, model development, the conversion costs used in the model of this research are introduced by Dong et al. The production cost-tolerance models proposed by Dong et al. are robust. This means that those production costs are not sensitive to changing in the costs of materials, labors, supplies, machines, power and so on. As a result, the conversion cost coefficients of the model of this research, model OTA, are not studied in their effects on the optimum solution. Furthermore, the unit costs of the scrap, reworking and inspection are not studied in their effects because the expected scrap and reworking costs are less than 0.001 percent of the conversion cost, and the expected inspection cost is about five percent. Another major reason for not studying the effect of the expected inspection cost was because it was measured as a percent of the conversion cost that was robust. All of the quality loss coefficients (6 coefficients) and all of the constraints (18 constraints) are studied in their effects. All quality loss coefficients and constraints are chosen as factors for applying experimental design to sensitivity analysis. Since determining an optimum solution of model OTA takes at least six to eight hours, Plackett-Burman Design with 2 levels factorial design in MINITAB is used to screen significant single factors. The significant factors are continued for study of two-factor interaction effects. Values for low and high levels are selected to be extreme conditions (in their effects on total cost) while also resulting in feasible solutions. These values are determined from preliminary study by trial and error. The chosen values are shown in table E-10. The 28 runs of the experiment for screening the significant single factors used in this research are sufficient for study 24 factors. The total cost optimized from model OTA for each run of the experiment is also shown in table E-10. The effects, coefficients and p-values from the analysis are shown in table E-11. A normal probability plot of the standardized effects with significance level equal to 0.05 is shown in table Figure E-1. The results from analyzing factorial design for screening significant factors are below:

1. Constraint 11 (constraint associated with the allowable maximum gap standard deviation) has a negative main effect, as expected, because the optimum value(s) of semi-tolerance zone(s) is (are) decreased when the allowable maximum process standard deviation of the gap is decreased. As a result, the total cost increases. Constraint 11 has the greatest effect, so it is chosen for studying two-factor interaction effects.

2. Constraints 21, 22, 23, 24, 25, and 26 (constraints associated with allowable ranges for semi-tolerance zones) have negative main effects. This results from the concept, already described in appendix F, used in the sensitivity analysis. Constraint 24 has the highest magnitude of the effect compared with the other in the group. Because its p-value, 0.048, was less than the specified significance level 0.05, it is chosen for study in two-factor interaction effects.
3. Constraints 31, 32, 33, 34, and 35 (constraints associated with the minimum specified process capability indices) have positive effects while constraint 36 has a negative. Based on the chosen original condition, the minimum requirement for process capability index for a part with inspection strategy starts to have a positive effect when the decreasing rate of the conversion cost is smaller than the increasing rate of the expected quality loss. From the example, part 1 and 2 are inspected whereas part 3 is not. In fact, the semi-tolerance zone for a part with non-inspection strategy does not affect the expected quality loss whereas it affects the conversion cost. Since the lower semi-tolerance zone for part 3 is already set at the allowable maximum value, the upper instead of the lower increases. As a result, its effect is negative. All constraints associated with the minimum requirements for process capability indices are not significant for studying in two-factor interaction effects.
4. Constraints 41, 42 and 43 (constraints associated with allowable ranges for standard deviations) have positive effects as expected based on the concept of the relationship between the process standard deviation and the tolerance as described in appendix F. All three constraints are significant at the significance level equal to 0.05. Therefore, they are studied in two-factor interaction effects in the next stage.

5. Based on the model concepts used in this research, the specified minimum proportions of conforming units for both sides of the product start to have positive effects when the decreasing rates of the conversion costs are smaller than the increasing rates of the expected quality losses. Since the decreasing rates of the conversion costs for both sides of the product are smaller than the increasing rates of the expected quality losses for the optimum solution for the chosen original condition, constraints 51 and 54 have positive effects. These constraints are not chosen for studying in two-factor interaction effects because they are not significant at the level of significance at 0.05.

Determining Significant Two-Factor Interaction Effects

From the first stage that deals with screening the significant single factors, constraint 11, 24, 41, 42 and 43 are chosen for further study in two-factor interaction effects. 2^{5-1} design is applied for determining the two-factor interaction effects. The generator for the selected design is factor 43 = factors 11*24*41*42. The values for the low and high levels of the factors along with the total cost from optimization are shown in table E-12.

Firstly, all of the main and two-factor interaction effects are estimated as shown in table E-13. In addition, a normal probability plot of the effects is shown in Figure E-2. The results from the preliminary study cannot calculate the p-values of the coefficients. However, it shows that the two-factor interaction effects of 11*24, 24*41, 24*42, and 24*43 are far smaller than the effects of the rest. Therefore, the value for each of these four effects is not calculated in the next stage in order to pool their error contributions in analysis of variances.

The effects, coefficients and the p-values for the second stage of study in two-factor interaction effects are shown in table E-14, and the normal probability plot of the standardized effects is shown in figure E-3. The magnitudes of the effects of factors 41*42, 41*43 and 42*43 are about 10 percent of that of factor 11, those of factors 11*41, 11*42, and 11*43 are greater than 25 percent; and those of factors 24, 41, 42, and 43 are greater than 50 percent. As a result, factors 41*42, 41*43 and 42*43 are determined to not be practical significant factors although they had p-values less than 0.05 while other factors are determined to be practical significant factors.

Conclusions for Sensitivity Analysis

Since the results from sensitivity analysis depend on the optimum solution for the chosen original condition, the conclusions and suggestions from the sensitivity analysis based on the chosen original condition for this example are expressed as follows:

1. The allowable maximum process standard deviation of the gap for the constraint associated with the gap standard deviation has the greatest effect on the total cost for optimizing tolerance allocation. Increasing the allowable maximum process standard deviation of the gap makes the optimum semi-tolerance zone(s) increase(s) resulting in reducing the total cost; its effect is negative. Therefore, any change even in a small amount of the allowable maximum process standard deviation of the gap needs to be considered carefully because it significantly affects the total cost of the product.
2. The allowable maximum process standard deviations of part 1, 2 and 3 have the second, third and fourth greatest effects, respectively, on the total cost. According to the concept of the relationship between the process standard

deviation and the tolerance used in this sensitivity analysis, increasing the allowable maximum process standard deviation(s) make(s) the total cost increase. Since each of them affects the total cost about fifty percent of the effect of the allowable maximum process standard deviation of the gap, each of them also needs to be carefully considered in its change.

3. The two-factor interaction effects of both the allowable maximum process standard deviation of the gap and the allowable maximum process standard deviation of part 1, part 2 or part 3 are significant. The allowable maximum process standard deviation of the gap itself has a negative effect whereas the allowable maximum process standard deviations of part 1, 2 and 3 have positive effects. Since the magnitude of the effect of the allowable maximum process standard deviation of the gap is about two times of that of each part, these two-factor interactions have negative effects on the total costs. This analysis explicitly shows that not only the single factors (the allowable maximum process standard deviations of the gap, part 1, part 2 and part 3) but also the two- factor interactions between the allowable maximum process standard deviations of the gap and part 1, the gap and part 2, and the gap and part 3 significantly affect the total cost. A condition with simultaneously changing in both the allowable maximum process standard deviation of the gap and that of any one of the parts needs to be considered in the effects of the gap, the part, and both the gap and the part.
4. The allowable maximum value for the upper semi-tolerance zone of part 1 is the single factor that has the smallest significant effect on the total cost

compared with other significant single factors. It has a negative effect on the total cost. The result suggests that increasing the allowable maximum value for the upper semi-tolerance zone of part 1 can reduce the total cost. This result confirms the critical contribution of optimizing tolerances.

5. The rest of the allowable maximum values for the semi-tolerance zones has negative effects that support the critical contribution of optimizing tolerances as well. Based on the chosen original condition, they have small magnitudes in the effects on the total cost for optimizing tolerance allocation. However, changing in these values could significantly affect the total cost for other original conditions especially for those with the optimum semi-tolerance zones set at the maximum values with the semi-tolerance zones of the gaps still having long lengths available for some more tolerances of the parts.

6. The quality loss coefficients for the lower side of part 1, and for the upper side of part 3 and 2 have positive effects that are ranked 6, 7 and 9 from all 24 factors, and the rest of the quality loss coefficients have positive effects as well. The effect of each of the quality loss coefficients is not statistically significant at the significance level equal to 0.05. However, the quality loss coefficient for the lower side of part 1 is almost significant. For some conditions and/or some products with high percentages of the expected quality losses in the total costs, these coefficients can significantly affect the total costs for optimizing tolerance allocation. For those cases, the effects of changing the quality loss coefficients need to be considered carefully.

7. The minimum requirements for process capability indices do not significantly affect the total cost for optimizing tolerance allocation. This results from the length of the optimum semi-tolerance zones for the chosen original condition being about 4.5 to 5.5 times of the standard deviations. Therefore, changing the minimum requirements for the process capability indices from 3.0 to 2.75 or 3.25 times of the standard deviations does not significantly affect the total cost. Nevertheless, for some conditions and/or some products with the increasing rates for some costs due to changing the minimum requirements for the process capability indices greater than the decreasing rates of the rest can significantly affect the total cost for optimizing tolerance allocation. As a result, specifying the minimum requirements for the process capability indices for those cases significantly affects the total costs of the products without requiring any action for improving the product quality. This is the critical contribution of optimizing tolerance allocation. An example demonstrating the effects of the minimum requirements for the process capability indices on the total cost for optimizing tolerance allocation is shown in table E-16. Its information for the chosen original condition can be seen in table E-15. The results from table E-16 show that for the conditions with the increasing rates of the expected quality losses greater than the decreasing rates of the conversion costs, those optimum semi-tolerance zones are equal to the specified minimum values of the indices. In contrast, for the conditions with the increasing rates of the expected quality losses smaller than the decreasing rates of the conversion costs, those optimum semi-tolerance zones are greater

than the specified minimum values. As can be seen in table E-16, the constraint associated with the minimum requirement for process capability index for the upper side of part 3 is already binding when the minimum value of the index is 3.0. And then, the constraints associated with the minimum requirement for process capability indices for the lower sides of part 1 and 2, the lower of part 3, and the upper of part 1 and 2 become binding in succession.

8. The specified minimum proportions of conforming units for the product do not significantly affect the total cost for optimizing tolerance allocation for the chosen original condition. As already mentioned, the actual dimension of the gap depends on the produced dimensions of the parts and envelope. In addition, the number of alternatives for the dimensions of the parts and envelope that could be assembled into a specified length of the gap is very large. Since the gap also has allowable ranges in lengths for the lower and the upper sides of its nominal size, there are several combinations with many alternatives among the dimensions of the parts and envelope for producing a conforming unit of the product. As a result, the specified minimum proportions of conforming units for the product have the smallest effects compared with those for other types of the constraints. However, for a product with low quality level, the specified minimum proportion(s) of conforming units for the product could affect(s) the total cost for optimizing tolerance allocation as can be seen in the example for verifying the effects of changing constraints 51 and 54 in table E-8.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Optimizing tolerance allocation deals with determining the optimum tolerances for the parts (components) that satisfy the specified tolerances for the envelope and finished product. It can reduce cost of a product without any investment or improvement in the quality of the product or the process while the product still satisfies all the specified requirements. There are several major limitations and conditions (see the list in the next paragraph) in applying a current model for optimizing tolerance allocation to industries. Therefore, the purpose of this research is to develop a model for optimizing tolerance allocation that can eliminate those limitations and conditions in application.

Model OTA proposed in this research can be applied to optimizing tolerance allocation with normal distributions that have the following characteristics: (1) the process mean of each part is either equal or unequal to the nominal size, (2) there is possible asymmetry in the upper and the lower quality loss coefficients, minimum requirements for the process capability indices, specified minimum semi-tolerance zones and/or specified minimum proportions of conforming units for the finished product, and (3) each part can independently choose the optimal inspection strategy among non-inspection strategy (NI), 100% inspection without reworking strategy (IWR) and 100% inspection with imperfect reworking strategy (IIR). In addition, model OTA can optimize tolerance allocation with either the specified minimum requirements for the process capability indices or the specified minimum proportions of conforming units because this research also proposes a procedure for transforming the specified minimum proportion of conforming units to the specified minimum requirement for the process capability indices.

Contributions of the Research

Since the appropriate approaches for developing the model proposed in this research avoid the significant assumptions and approximations of current models by dealing with those conditions exactly, the model can represent the product's behavior far better than the current models. This means that the model proposed in this research can allow a level of control nearer the global optimum resulting in lower cost than those determined from the current models. Moreover, the approaches used in this research may be applied to optimizing tolerance allocation for other types of products, such as non-manufacturing problems. This research proposes additional formulas of C_{pm} that independently measure the capability of a process for manufacturing a part with a dimension falling between the lower specification limit and the nominal size, or between the nominal size and the upper limit.

The critical contribution of this research is proposing a model for optimizing tolerance allocation with all of the characteristics listed in chapter 1 (except skewed distributions) that can be solved by using common off-the-shelf software. This makes optimizing tolerance allocation more practical and understandable for users. Microsoft Excel is the example of the common-off-the-shelf software chosen in this research because it is widely used in almost every industry.

Another purpose of this research is applying design of experiments to sensitivity analysis. It can reduce the number of experiments while it can evaluate the effects of cost coefficients in the objective function and those of constraints to the total cost with a specified level of significance. Model OTA of this research can optimize tolerance allocation with any number of the parts having normal distributions (the number of the parts does not matter for the capability of the model). However, the capability of applying model OTA to a product for inspection strategies depends on the speeds and capabilities of the computers because simulating the proportion of conforming units for the finished product takes very long time.

Methodologies and Techniques used in The Research

The fourth order polynomial production cost-tolerance model for face milling studied in terms of the production cost increase compared with the cost of casting processes vs. tolerance introduced by Dong et al. was used as the conversion cost for model OTA. It is chosen because it has small fitting error for tolerance smaller than 0.35 mm for most of products with parts needing machining processes and it is believed to be robust.

Quadratic quality loss function is used in the objective function of model OTA because it is simpler than polynomial but more reasonable than the step loss function. This research deals with optimizing semi-tolerance zones of the parts assembled in an envelope based on the assumption that the process standard deviation of each part has an increasing linear relationship with its tolerance. In order to optimize tolerance allocation by using Microsoft Excel, the formulas in terms of integrations of the expected quality losses with normal distributions must be transformed to the formulas with truncated normal distributions that are not in terms of function integrations.

Model OTA is solved by genetic algorithm in Evolver, and Add-In for Microsoft Excel. For a case with at least one part and/or the envelope inspected, the gap dimension of the finished product does not have a nice distribution. As a result, simulation is needed to simulate the proportion of conforming units for each side of the finished product. Model verification is also performed in order to ensure that the concepts used in model OTA are correct. In order to reduce the number of experimental runs, Plackett-Burman Design is used for screening significant single factors and then only the significant single factors are continued for study in two-factor interactions based on a fractional factorial design.

Learning from the Research

Since the value of the total cost is more than one thousand times the value with 3 decimal places (but not an integer) of the semi-tolerance zone for each side of each part, the optimum values of the semi-tolerance zones are not sensitive to the total cost. As a result, determining appropriate conditions for running Evolver is critical to consistency of the optimum solutions. The appropriate conditions for running Evolver for numerical examples in this research are described in Appendix G.

In addition, conversion cost critically affects the optimum solution (the total cost and the semi-tolerance zones). Therefore, the concept used to divide the conversion cost into two components, one for each of the upper and the lower semi-tolerance zones, needs to be carefully followed in order to receive the consistent optimum solutions.

Recommendations

The solution determined from model OTA proposed in this research depends on the chosen inspection strategy for each part. Developing a model for optimizing tolerance allocation that includes the approach for choosing the optimal inspection strategies for the parts is a worthwhile research. Due to generating random numbers of a heuristic approach, simultaneously determining the optimal inspection strategies and semi-tolerance zones is more likely to mean the model cannot reach the optimal inspection strategies. This results when the model stops searching before it can reach the optimal inspection strategy combination although the chosen stopping condition is already set at the condition with very small difference in very large successive numbers of trials. Therefore, the models being proposed in the future may consider optimizing semi-tolerance zones for each combination of the inspection strategies and then choosing the optimal combination for inspection strategies with the least total cost at the final stage.

There are some current models dealing with optimizing tolerance allocation for only one-sided tolerances for Weibull and Gamma distributions whereas many products need asymmetrical two-sided tolerances for the quality characteristics with skewed distributions. Therefore, developing a model with all characteristics as those for model OTA that can optimize allocating asymmetrical two-sided tolerances for skewed distributions, such as Beta distributions that can be left or right skewed distributed, is very useful for real application. If products with skewed distributions are considered, those distributions must be transformed to normal distributions (one choice is using Box-Cox transformation) in order to keep using C_{pm_L} and C_{pm_U} in the constraints associated with the minimum requirements for process capability indices. If not, other process capability indices that are developed for non-normal are needed.

The C_{pm_L} and C_{pm_U} that are introduced by this research would be useful and additional option for industrial users. They can measure the capability of the process potentially more accurately than C_{pm} because they independently measure the capability of the process for below and above the nominal size. Therefore, they can overcome the pitfall due to the target value (nominal size for this research) being not equal to the midpoint of the specification interval. In order to ensure their properties for correct application to real industry, these indices and their expected values need to be mathematically verified. This is a potentially interesting future research.

All of the constraints of model OTA in this research are hard constraints that must be satisfied. Considering some soft constraints (with penalty functions for the constraints that the customers allow some ranges for violating the constraints with some penalty costs) may be able to reduce the total costs for some products. Therefore, proposing the model with hard and soft constraints is another choice for improving the optimum solution and more practical in real application.

The conversion cost, the expected inspection, reworking and scrap costs are not studied in their sensitivities to the total cost for model OTA in this research. There are many products with manufacturing processes different from those studied by Dong et al; as a result, the conversion costs need to be studied in their sensitivities to the total cost. For products with the expected inspection, reworking and/or scrap costs whichare not small fractions in the total cost and are not measured as percent of the conversion cost, they also need sensitivity analysis. Moreover, the significant two-factor interactions of these cost coefficients should be added into the objective function in order to increase the accuracy of the total cost.

Some products may have multiple local optimum solutions with ranges for economic solutions. This means that they have ranges for semi-tolerance zones that have small differences in those total costs from the global optimum total costs. Determining the ranges of economic semi-tolerance zones by studying their total cost surfaces is a major extension for this research.

Some solutions may have inconsistent optimum semi-tolerance zones. This means that different starting points give the greater semi-tolerance zones on different sides for the same parts with the same conditions. This could happen for products when the optimum semi-tolerance zones are not sensitive to the total costs and/or for products having large numbers of the parts to be assembled in the envelopes. For those cases, ability to study their total cost surfaces and/or determine the economic ranges for the semi-tolerance zones is critically needed.

Model OTA assumes that the process standard deviation of each part has an increasing linear relationship with its tolerance. For some products, their process standard deviations may have other relationships instead of linear relationships with the tolerances or the standard deviation of each part may need to be optimized independently. Therefore, the process standard deviations may need sensitivity analyses and if at least one of them has significant two-factor interactions with each other or with other cost coefficients, that (those) two-factor interaction(s) need(s) to be added in the objective function as well.

APPENDICES

Appendix A

A Full Explanation Of The Dong Model

Dong et al. introduced various production cost-tolerance models for tolerance allocation, both hybrid and polynomial. For hybrid models, the researchers proposed (1) combined reciprocal power and exponential function, (2) combined linear and exponential function, and (3) fourth-order B-Spline curve, while for the polynomial models, cubic, fourth-order, and fifth-order are suggested. They determined the optimal model parameters using least square approximation for empirical production cost-tolerance data curves for (1) generic relation for typical production processes studied by Dieter (1983), (2) die casting, (3) investment casting, (4) true position of holes, (5) face milling, (6) turning on lathe, (7) rotary surface grinding, and (8) internal grinding. All of the production cost-tolerance models introduced by them as well as the existing ones, (1) reciprocal squared, (2) reciprocal, (3) exponential, (4) reciprocal power, and (5) reciprocal power and exponential hybrid, were ranked based on fitting errors. Overall, beginning with the best, the models were ranked as first, the fifth-order polynomial; second, the fourth-order polynomial; and third, the combined reciprocal power and exponential function model.

Face milling is popularly used for most manufactured products with mild to high quality levels of surface finishes, and the parts of the numerical examples constructed in this research have tight or medium tolerance zone (≤ 0.1 or 0.1 to 0.35 mm, respectively) according to Dong et al's classification. The fourth-order polynomial function model was chosen for this research because it ranked third from eleven models for face milling and because it does not require a large number of terms or a complicated function as the fourth-order B-Spline Curve.

The users only need to follow the procedure being proposed below for applying Dong et al's models.

(1) The manufacturers collect and plot the sufficient raw data in terms of actual production costs for the processes used (measured in the units of their currencies) for producing one unit of the product vs. tolerances. The chosen processes that can be applied to Dong et al's models are (1) generic relation for typical production processes studied by Dieter (1983), (2) die casting, (3) investment casting, (4) true position of holes, (5) face milling, (6) turning on lathe, (7) rotary surface grinding, and (8) internal grinding.

(2) Each of the cost functions proposed by Dong et al., C_{Dong}, is the production cost increase for the chosen process compared with the production cost of the casting process vs. tolerance. The most appropriate production cost function, C^{*}_{Dong}, is one of the first four best-fit costs with uncomplicated function chosen from (1) reciprocal squared, (2) reciprocal, (3) exponential, (4) reciprocal power, (5) reciprocal power and exponential hybrid, (6) combined linear and exponential, (7) cubic polynomial, (8) combined reciprocal power and exponential, (9) fourth-order B-Spline, (10) fourth-order polynomial or (11) fifth-order polynomial function. The raw production cost, C_A, can be approximated by applying the formula:

$$C_A = \left\{ C^{*}_{Dong} * \frac{1}{100} + 1 \right\} * C_M \ \_\_(A-1) \text{ where}$$

C_M = multiplier of conversion cost for calculating the raw conversion cost for producing one unit of each part. It is similar to the unit cost of casting process that is used as the reference for determining the relative cost for other processes in Dong et al's cost model

(3) Choosing the most appropriate production cost function needs sufficient experimental data for the entire range of the tolerance that was studied by Dong et al. or at least for the entire range of the allowable tolerance range of the product.

The conversion cost, which is the fourth-order polynomial for face milling proposed by Dong et al., for producing one unit of each part is:

$$C_A = \left\{ C_{Dong} * \frac{1}{100} + 1 \right\} * C_M \text{, where}$$

$$C_{Dong} = A + B*\Delta + C*\Delta^2 + D*\Delta^3 + F*\Delta^4 \quad \_\_(A-2) \text{, where}$$

A = fixed conversion cost for producing one unit of each part

B = conversion cost coefficient due to linear function of tolerance for producing one unit of each part

C = conversion cost coefficient due to quadratic function of tolerance for producing one unit of each part

D = conversion cost coefficient due to cubic polynomial of tolerance for producing one unit of each part

F = conversion cost coefficient due to fourth-order polynomial of tolerance for producing one unit of each part

Δ = tolerance for each part

Appendix B

Expected Quality Losses for Truncated Normal Distributions

Let $X \sim N(\mu, \sigma^2)$, i.e., X has a normal distribution with mean μ and variance σ^2.

Then X has density function

$$f_X(x) = \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right), \text{ where}$$
$$\phi(x) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}x^2\right].$$

Each part for model OTA has a potentially different normal distribution X_i, but for notational convenience the subscript i will be suppressed unless required for clarity.

The truncated normal distribution needs to be applied in order to determine the expected quality loss by using Microsoft Excel. A random variable T_D is said to have a doubly truncated normal distribution with lower truncation point A and upper truncation point B if it has density function

$$f_{T_D}(x) = \frac{f_X(x)}{\Phi(B^*) - \Phi(A^*)} \quad \textit{for } A \le x \le B$$

where

$$\Phi(x) = \int_{-\infty}^{x} \phi(u)\,du$$
$$A^* = \frac{A-\mu}{\sigma}$$
$$B^* = \frac{B-\mu}{\sigma}$$

Letting

$$P\left[A^*, B^*\right] = \Phi(B^*) - \Phi(A^*)$$
$$= \int_{A^*}^{B^*} \phi(x)\,dx$$

It follows that

$$f_{T_D}(x) = \frac{f_X(x)}{P_{[A^*,B^*]}} \quad for\ A \le x \le B$$

The mean μ_{T_D} and the variance $\sigma^2_{T_D}$ of the doubly truncated normal distribution are (see Johnson and Kotz (1970))

$$\mu_{T_D} = \mu + \frac{\phi(A^*) - \phi(B^*)}{P_{[A^*,B^*]}}\sigma \quad \_\_(B-1)$$

$$\sigma^2_{T_D} = \left[1 + G[A^*,B^*] - \left\{\frac{\phi(A^*) - \phi(B^*)}{P[A^*,B^*]}\right\}^2\right] * \sigma^2 \quad \_\_(B-2)$$

where

$$G[A^*,B^*] = \frac{A^*\phi(A^*) - B^*\phi(B^*)}{P_{[A^*,B^*]}}$$

We will also use the fact that $E\left[(X-N)^2\right] = \sigma^2 + (\mu - N)^2 \quad \_\_(B-3)$ (see page 36)

For a part with IWR or IIR (excluding the effect of imperfect reworking), its quality loss function is:

$$QL = \begin{cases} K_L(x-N)^2 & for\ N - \Delta_L \le x \le N \\ K_U(x-N)^2 & for\ N \le x \le N + \Delta_U \end{cases}.$$

The expected quality loss for each part with IWR or IIR (excluding the effect of imperfect reworking) is

$$E(QL) = \int_{N-\Delta_L}^{N} K_L(x-N)^2 f_X(x)dx + \int_{N}^{N+\Delta_U} K_U(x-N)^2 f_X(x)dx$$

$$= P_{[(N-\Delta_L)^*,N^*]} K_L \int_{N-\Delta_L}^{N} (x-N)^2 f_{T_{DL}}(x)dx + P_{[N^*,(N+\Delta_U)^*]} K_U \int_{N}^{N+\Delta_U} (x-N)^2 f_{T_{DU}}(x)dx$$

$$= P_{[(N-\Delta_L)^*,N^*]} K_L E\left[(T_{DL} - N)^2\right] + P_{[N^*,(N+\Delta_U)^*]} K_U E\left[(T_{DU} - N)^2\right] \quad \_\_(B-4)$$

where

$$(N-\Delta_L)^* = \frac{N-\Delta_L-\mu}{\sigma}$$

$$(N+\Delta_U)^* = \frac{N+\Delta_U-\mu}{\sigma}$$

$$N^* = \frac{N-\mu}{\sigma}$$

T_{D_L} is a doubly truncated normal distribution with lower truncation point $N-\Delta_L$ and upper truncation point N, and T_{D_U} is a doubly truncated normal distribution with lower truncation point N and upper truncation point $N+\Delta_U$. Using equation $(B-3)$, equation $(B-4)$ can be written as

$$E(QL) = P_{\left[(N-\Delta_L)^*, N^*\right]} K_L\left[\sigma^2_{T_{D_L}} + \left(\mu_{T_{D_L}} - N\right)^2\right]$$
$$+ P_{\left[N^*, (N+\Delta_U)^*\right]} K_U\left[\sigma^2_{T_{D_U}} + \left(\mu_{T_{D_U}} - N\right)^2\right] \text{\_\_}(B-5)$$

where (from equation $(B-1)$ and $(B-2)$)

$$\mu_{T_{D_L}} = \mu + \frac{\phi((N-\Delta_L)^*) - \phi(N^*)}{P_{\left[(N-\Delta_L)^*, N^*\right]}}\sigma$$

$$\mu_{T_{D_U}} = \mu + \frac{\phi(N^*) - \phi((N+\Delta_U)^*)}{P_{\left[N^*, (N+\Delta_U)^*\right]}}\sigma$$

$$\sigma^2_{T_{D_L}} = \left[1 + G((N-\Delta_L)^*, N^*) - \left\{\frac{\phi((N-\Delta_L)^*) - \phi(N^*)}{P_{\left[(N-\Delta_L)^*, N^*\right]}}\right\}^2\right]\sigma^2$$

$$\sigma^2_{T_{D_U}} = \left[1 + G(N^*, (N+\Delta_U)^*) - \left\{\frac{\phi(N^*) - \phi((N+\Delta_U)^*)}{P_{\left[N^*, (N+\Delta_U)^*\right]}}\right\}^2\right]\sigma^2$$

For a part with NI, its quality loss function is:

$$QL = \begin{cases} K_L(x-N)^2 & for -\infty < x \le N \\ K_U(x-N)^2 & for\ N \le x < \infty \end{cases}$$

The expected quality loss for each part with non−inspection strategy is

$$E(QL) = \int_{-\infty}^{N} K_L(x-N)^2 f_X(x)\,dx + \int_{N}^{\infty} K_U(x-N)^2 f_X(x)\,dx$$

$$= P_{[-\infty,N^*]} K_L \int_{-\infty}^{N} (x-N)^2 f_{T_{S_L}}(x)\,dx + P_{[N^*,\infty]} K_U \int_{N}^{\infty} (x-N)^2 f_{T_{S_U}}(x)\,dx$$

$$= P_{[-\infty,N^*]} K_L E\left[(T_{S_L} - N)^2\right] + P_{[N^*,\infty]} K_U E\left[(T_{S_U} - N)^2\right] \quad \_\_(B-6)$$

T_{S_L} is a singly truncated normal distribution with upper truncation point N (nominal size for each part), and T_{S_U} is a singly truncated normal distribution with lower truncation point N. Using equation $(B-3)$, equation $(B-6)$ can be written as

$$E(QL) = P_{[-\infty,N^*]} K_L \left[\sigma^2_{T_{S_L}} + (\mu_{T_{S_L}} - N)^2\right] + P_{[N^*,\infty]} K_U \left[\sigma^2_{T_{S_U}} + (\mu_{T_{S_U}} - N)^2\right] \_\_(B-7)$$

where (from equation $(B-1)$ and $(B-2)$)

$$\mu_{T_{S_L}} = \mu + \frac{-\phi(N^*)}{P_{[-\infty,N^*]}}\sigma$$

$$\mu_{T_{S_U}} = \mu + \frac{\phi(N^*)}{P_{[N^*,\infty]}}\sigma$$

$$\sigma^2_{T_{S_L}} = \left[1 + G(-\infty, N^*) - \left\{\frac{-\phi(N^*)}{P_{[-\infty,N^*]}}\right\}^2\right]\sigma^2$$

$$\sigma^2_{T_{S_U}} = \left[1 + G(N^*, \infty) - \left\{\frac{\phi(N^*)}{P_{[N^*,\infty]}}\right\}^2\right]\sigma^2$$

Appendix C

Process Standard Deviation has An Increasing Linear Relationship with Tolerance

The assumption about the relationship between the process standard deviation and the tolerance for each part details below:

Numerical examples of this research assume that the manufacturer has collected the empirical data between the appropriate process standard deviations and the tolerances of the parts. Regression analyses for the relationships between the process standard deviations and the tolerances have been performed from the empirical data. We assume that the process standard deviation of each part has an increasing linear relationship with the tolerance of that part according to the empirical studies of the product and its process as shown in Figure C-1.

Process Standard Deviation

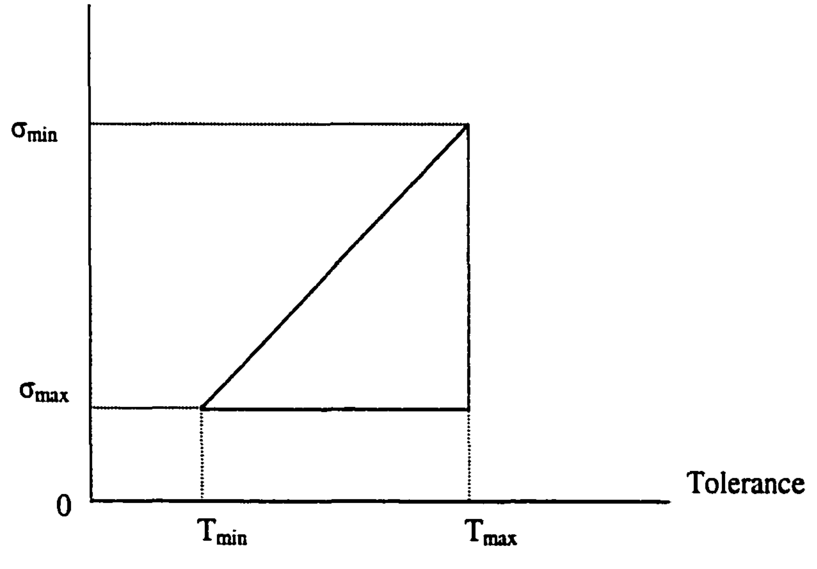


Figure C-1 Process Standard Deviation has an increasing linear relationship with Tolerance

Letting

$\sigma_{\min}$ = the allowable minimum process standard deviation of each part that depends on the capability of the machine and/or process used to manufacture the part

$\sigma_{\max}$ = the allowable maximum process standard deviation of each part depending on the economic condition that depends on the capability of the machine and/or the process used to manufacture the part.

$T_{\min} = \Delta_{MinA_L} + \Delta_{MinA_U}$

= the allowable minimum value of the tolerance for each part based on the capability of the machine and/or process used to manufactured the part

$T_{\max} = \Delta_{Max_L} + \Delta_{Max_U}$

= the allowable maximum value of the tolerance for each part based on the capability of the machine and/or process used to manufactured the part

where

Δ_{MinA_L} = the allowable minimum value of the lower semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured that part

Δ_{MinA_U} = the allowable minimum value of the upper semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured that part

Δ_{Max_L} = the allowable maximum value of the lower semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured the part

Δ_{Max_U} = the allowable maximum value of the upper semi-tolerance zone for each part

based on the capability of the machine and/or process used to manufactured the part

Finally, the process standard deviation that has an increasing linear relationship with the tolerance can be expressed as:

$$\sigma = \sigma_{\min} + \left\{ \frac{\sigma_{\max} - \sigma_{\min}}{(\Delta_{\max L} + \Delta_{\max U}) - (\Delta_{MinA_L} + \Delta_{MinA_U})} \right\} * \{(\Delta_L + \Delta_U) - (\Delta_{MinA_L} + \Delta_{MinA_U})\}$$

where

σ = the optimum standard deviation for each part. It affects the expected quality loss of the objective function in the model OTA

Δ_L = the lower semi-tolerance zone being optimized

Δ_U = the upper semi-tolerance zone being optimized

Appendix D

Practical Simulation Concept

For a practical simulation concept, one accepted value for the dimension for each part and that for the envelope are generated at each generation. If the generated dimension for a part is within the specification limits, it is accepted for assembling a finished product. A generated dimension outside the specification limits is rejected, and new dimensions generated for that part until it is conforming to the limits. A generated dimension outside the specification limits can represent the actual outcome for both IWR and IIR. Following successful generation of a conforming dimension for the first part, the dimensions for the remaining parts and the envelope are generated as above. .

Next, the conforming value for the generated dimension for envelope minus the sum of dimensions for parts is compared to the specification limit of the gap. The assembly will be accepted if the gap dimension falls within the specification limits; otherwise, it is rejected. This procedure is repeated until sufficient assemblies have been generated for the desired accuracy of the simulation. Counts of parts and assemblies conforming and nonconforming are used to estimate proportions of parts and assemblies acceptable in the manufacturing process.

Appendix E

Data and Results for Model Verifications and Sensitivity Analysis

Table E-1 Data for an Original Condition for a Product for Verifying Changes in Means

Information	Side of Part 1		Side of Part 2		Side of Part 3		Side of Envelope		Side of Gap	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Allowable Minimum Semi-Tolerance Zone (mm)	0.055	0.055	0.055	0.055	0.055	0.055	N/A	N/A	N/A	N/A
Allowable Maximum Semi-Tolerance Zone (mm)	0.085	0.085	0.085	0.085	0.085	0.085	N/A	N/A	N/A	N/A
Allowable Minimum Semi-Tolerance Zone Based on The Capability of the Machine (mm)	0.019	0.019	0.019	0.019	0.019	0.019	N/A	N/A	N/A	N/A
Specified Minimum C*pm Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone	4	4	4	4	4	4	N/A	N/A	N/A	N/A
Specified Minimum Proportion of Conforming Part (Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone)	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	3	3
Fixed Conversion Cost	280.7	280.7	280.7	280.7	280.7	280.7	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 1st Order	2407	2407	2407	2407	2407	2407	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 2nd Order	282.3	282.3	282.3	282.3	282.3	282.3	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 3rd Order	45960	45960	45960	45960	45960	45960	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 4th Order	106100	106100	106100	106100	106100	106100	N/A	N/A	N/A	N/A
Quality Loss Coefficient, K	20340	13560	15570	10380	12320	18480	N/A	N/A	N/A	N/A
Optimum Semi-Tolerance Zone (mm)	0.07	0.085	0.064	0.083	0.079	0.059	0.075	0.075	0.16	0.16

Table E-1 Data for an Original Condition for a Product for Verifying Changes in Means

Information	Part 1	Part 2	Part 3	Envelope	Gap
Nominal Size (mm)	50.455	40.725	38.75	130.1	0.17
Process Mean of the Dimension (mm)	50.459	40.729	38.746	130.106	0.172
Minimum allowable Process Standard Deviation of the Dimension (mm)	0.012	0.012	0.012	N/A	N/A
Maximum allowable Process Standard Deviation of the Dimension (mm)	0.0156	0.0156	0.0156	N/A	0.029
Optimum Process Standard Deviation of the Dimension (mm)	0.0151909	0.0149727	0.0147273	0.013	0.029
Mean Offsets from the Nominal Size (Numbers of Standard Deviation)	0.2633154	0.2671524	0.2716049	0.4615385	0.0689655
Inspection Strategy	IIR	IWR	NI	NI	NI
Multiplier for Determining Raw Conversion Cost	25	20	19	N/A	N/A
Inspection Cost (Measured as Percentage of Conversion Cost)	10	10	10	N/A	N/A
Scrap Cost (Measured as Percentage of Conversion Cost)	200	200	200	N/A	N/A
Reworking Cost (Measured as Percentage of Conversion Cost)	25	25	25	N/A	N/A

Optimum Total Cost ($) = 98.01929

Binding Constraint: 11, 24, 36

N/A : No Available Information

Table E-2 Model Verification Results: Effects of Process Mean Shifts on Semi-Tolerance Zones, Costs and Binding Constraints

Condition Changed / Values from Optimal Solution	Value(s) of Process Mean						
	$N_1<Mean_1$, $N_2<Mean_2$, $N_3>Mean_3$ (Original Condition)	$N_1 = Mean_1$	$N_1 > Mean_1$	$N_2 = Mean_2$	$N_2 > Mean_2$	$N_3 = Mean_3$	$N_3 < Mean_3$
Lower Semi-Tolerance Zone for Part 1 (mm)	0.07	0.079	0.085	0.069	0.07	0.069	0.07
Upper Semi-Tolerance Zone for Part 1 (mm)	0.085	0.078	0.07	0.085	0.085	0.085	0.085
Lower Semi-Tolerance Zone for Part 2 (mm)	0.064	0.063	0.064	0.074	0.083	0.063	0.064
Upper Semi-Tolerance Zone for Part 2 (mm)	0.083	0.082	0.083	0.074	0.064	0.083	0.083
Lower Semi-Tolerance Zone for Part 3 (mm)	0.079	0.079	0.079	0.079	0.079	0.07	0.059
Upper Semi-Tolerance Zone for Part 3 (mm)	0.059	0.059	0.059	0.059	0.059	0.07	0.079
Conversion Cost for Lower Side of Part 1 ($)	12.74970979	15.3250331	18.190694	12.88097516	12.74970979	12.88097516	12.74970979
Conversion Cost for Upper Side of Part 1 ($)	18.190694	15.46557093	12.74970979	18.19619472	18.190694	18.19619472	18.190694
Conversion Cost for Lower Side of Part 2 ($)	10.88375609	11.00934702	10.88375609	12.8735048	14.84261741	11.01460252	10.88375609
Conversion Cost for Upper Side of Part 2 ($)	14.84261741	14.98804714	14.84261741	12.87350482	10.88375609	14.847232	14.84261741
Conversion Cost for Lower Side of Part 3 ($)	14.69449378	14.69449378	14.69449378	14.69449378	14.69449378	12.78775467	10.98237891
Conversion Cost for Upper Side of Part 3 ($)	10.98237891	10.98237891	10.98237891	10.98237891	10.98237891	12.78775469	14.69449378
Expected Quality Loss for Lower Side of Part 1 ($)	1.512040686	2.363736951	3.507070812	1.50535396	1.512040686	1.50535396	1.512040686
Expected Quality Loss for Upper Side of Part 1 ($)	2.338046024	1.575820796	1.008027635	2.331267104	2.338046024	2.331267104	2.338046024
Expected Quality Loss for Lower Side of Part 2 ($)	1.116781596	1.106722736	1.116781596	1.751589219	2.62266835	1.111714736	1.116781596
Expected Quality Loss for Upper Side of Part 2 ($)	1.748445567	1.738198045	1.748445567	1.167726147	0.744521064	1.74331973	1.748445567
Expected Quality Loss for Lower Side of Part 3 ($)	2.020784213	2.020784213	2.020784213	2.020784213	2.020784213	1.345973236	0.848452151
Expected Quality Loss for Upper Side of Part 3 ($)	1.748445592	1.7381981	1.748445592	1.167750001	0.744649402	1.743317966	1.748445592
Total Cost for Part1	37.88456575	37.80923064	38.54955163	38.02155444	37.88456575	38.02155444	37.88456575
Total Cost for Part2	31.16438518	31.44224313	31.16438518	31.24106764	31.66634743	31.30324623	31.16438518
Total Cost for Part3	28.97033513	28.97033513	28.97033513	28.97033513	28.97033513	28.94044245	29.55650116
Grand Total Cost	98.01928606	98.2218089	98.68427194	98.23295721	98.52124831	98.26524312	98.60545209
Binding Constraints	11, 24, 36	11, 36	11, 21, 36	11, 24, 36	11, 24, 36	11, 24	11, 24, 33

Notation: $Mean_i$ = the process mean of the dimension for i^{th} part

Table E-2 Model Verification Results: Effects of Process Mean Shifts on Semi-Tolerance Zones, Costs and Binding Constraints (Continued)

Condition Changed / Values from Optimal Solution	Values of Process Means						
	$N_1=Mean_1, N_2=Mean_2$	$N_1=Mean_1, N_2>Mean2$	$N_1>Mean_1, N_2=Mean_2$	$N_1>Mean_1, N_2> Mean_2$	$N_1=Mean_1, N_3=Mean_3$	$N_1=Mean_1, N_3<_{Mean3}$	$N_1>Mean_1, N_3=Mean_3$
Lower Semi-Tolerance Zone for Part 1 (mm)	0.078	0.079	0.085	0.085	0.079	0.079	0.085
Upper Semi-Tolerance Zone for Part 1 (mm)	0.078	0.078	0.069	0.07	0.078	0.079	0.069
Lower Semi-Tolerance Zone for Part 2 (mm)	0.073	0.082	0.074	0.083	0.063	0.063	0.063
Upper Semi-Tolerance Zone for Part 2 (mm)	0.074	0.063	0.074	0.064	0.082	0.082	0.083
Lower Semi-Tolerance Zone for Part 3 (mm)	0.078	0.079	0.079	0.079	0.069	0.059	0.07
Upper Semi-Tolerance Zone for Part 3 (mm)	0.059	0.059	0.059	0.059	0.069	0.078	0.07
Conversion Cost for Lower Side of Part 1 ($)	15.46557163	15.3250331	18.19619472	18.190694	15.3250331	15.3250324	18.19619472
Conversion Cost for Upper Side of Part 1 ($)	15.46557164	15.46557093	12.88097516	12.74970979	15.46557093	15.32503241	12.88097516
Conversion Cost for Lower Side of Part 2 ($)	13.0120403	14.98804714	12.8735048	14.84261741	11.00934702	11.00934702	11.01460252
Conversion Cost for Upper Side of Part 2 ($)	12.87350683	11.00934702	12.87350482	10.88375609	14.98804714	14.98804714	14.847232
Conversion Cost for Lower Side of Part 3 ($)	14.85279084	14.69449378	14.69449378	14.69449378	12.94057754	10.97697064	12.78775467
Conversion Cost for Upper Side of Part 3 ($)	10.97697064	10.98237891	10.98237891	10.98237891	12.94057755	14.85279084	12.78775469
Expected Quality Loss for Lower Side of Part 1 ($)	2.355282727	2.363736951	3.496903082	3.507070812	2.363736951	2.372200646	3.496903082
Expected Quality Loss for Upper Side of Part 1 ($)	1.570188486	1.575820796	1.003570003	1.008027635	1.575820796	1.581467099	1.003570003
Expected Quality Loss for Lower Side of Part 2 ($)	1.745214191	2.607297068	1.751589219	2.62266835	1.106722736	1.106722736	1.111714736
Expected Quality Loss for Upper Side of Part 2 ($)	1.163484903	0.737815157	1.167726147	0.744521064	1.738198045	1.738198045	1.74331973
Expected Quality Loss for Lower Side of Part 3 ($)	2.014781138	2.020784213	2.020784213	2.020784213	1.336058181	0.844567662	1.345973236
Expected Quality Loss for Upper Side of Part 3 ($)	1.163507497	0.737973205	1.167750001	0.744649402	1.7381981	1.7381981	1.743317966
Total Cost for Part1	37.94973962	37.80923064	38.68537088	38.54955163	37.80923064	37.66874736	38.68537088
Total Cost for Part2	31.38284908	31.94243457	31.24106764	31.66634743	31.44224313	31.44224313	31.30324623
Total Cost for Part3	29.11139411	28.97033513	28.97033513	28.97033513	29.22130054	29.69650085	28.94044245
Grand Total Cost	98.44398282	98.72200035	98.89677364	99.1862342	98.47277431	98.80749134	98.92905955
Binding Constraints	11, 36	11, 36	11, 21, 36	11, 21, 36	11	11, 33	11, 21

Notation: $Mean_i$ = the process mean of the dimension for i^{th} part

Table E-2 Model Verification Results: Effects of Process Mean Shifts on Semi-Tolerance Zones, Costs and Binding Constraints (Continued)

Condition Changed / Values from Optimal Solution	Values of Process Means						
	$N_1>Mean_1, N_3<Mean_3$	$N_2=Mean_2, N_3=Mean_3$	$N_2=Mean_2, N_3<Mean_3$	$N_2>Mean_2, N_3=Mean_3$	$N_2>Mean_2, N_3<Mean_3$	$N_1=Mean_1, N_2=Mean_2, N_3=Mean_3$	$N_1>Mean_1, N_2>Mean_2, N_3<Mean_3$
Lower Semi-Tolerance Zone for Part 1 (mm)	0.085	0.069	0.069	0.069	0.07	0.078	0.085
Upper Semi-Tolerance Zone for Part 1 (mm)	0.07	0.085	0.085	0.085	0.085	0.078	0.07
Lower Semi-Tolerance Zone for Part 2 (mm)	0.064	0.074	0.074	0.083	0.083	0.073	0.083
Upper Semi-Tolerance Zone for Part 2 (mm)	0.083	0.074	0.074	0.063	0.064	0.073	0.064
Lower Semi-Tolerance Zone for Part 3 (mm)	0.059	0.069	0.059	0.07	0.059	0.069	0.059
Upper Semi-Tolerance Zone for Part 3 (mm)	0.079	0.069	0.079	0.07	0.079	0.069	0.079
Conversion Cost for Lower Side of Part 1 ($)	18.190694	12.88097516	12.88097516	12.88097516	12.74970979	15.46557163	18.190694
Conversion Cost for Upper Side of Part 1 ($)	12.74970979	18.19619472	18.19619472	18.19619472	18.190694	15.46557164	12.74970979
Conversion Cost for Lower Side of Part 2 ($)	10.88375609	12.8735048	12.8735048	14.847232	14.84261741	13.01204234	14.84261741
Conversion Cost for Upper Side of Part 2 ($)	14.84261741	12.87350482	12.87350482	11.01460252	10.88375609	13.01204235	10.88375609
Conversion Cost for Lower Side of Part 3 ($)	10.98237891	12.94057754	10.98237891	12.78775467	10.98237891	12.94057754	10.98237891
Conversion Cost for Upper Side of Part 3 ($)	14.69449378	12.94057755	14.69449378	12.78775469	14.69449378	12.94057755	14.69449378
Expected Quality Loss for Lower Side of Part 1 ($)	3.507070812	1.50535396	1.50535396	1.50535396	1.512040686	2.355282727	3.507070812
Expected Quality Loss for Upper Side of Part 1 ($)	1.008027635	2.331267104	2.331267104	2.331267104	2.338046024	1.570188486	1.008027635
Expected Quality Loss for Lower Side of Part 2 ($)	1.116781596	1.751589219	1.751589219	2.614979595	2.62266835	1.73886436	2.62266835
Expected Quality Loss for Upper Side of Part 2 ($)	1.748445567	1.167726147	1.167726147	0.741143158	0.744521064	1.159242907	0.744521064
Expected Quality Loss for Lower Side of Part 3 ($)	0.848452151	1.336058181	0.848452151	1.345973236	0.848452151	1.336058181	0.848452151
Expected Quality Loss for Upper Side of Part 3 ($)	1.748445592	1.167750001	1.167750001	0.741307462	0.744649402	1.159272715	0.744649402
Total Cost for Part1	38.54955163	38.02155444	38.02155444	38.02155444	37.88456575	37.94973962	38.54955163
Total Cost for Part2	31.16438518	31.24106764	31.24106764	31.80433451	31.66634743	31.52465449	31.66634743
Total Cost for Part3	29.55650116	29.22130054	29.55650116	28.94044245	29.55650116	29.22130054	29.55650116
Grand Total Cost	99.27043798	98.48392262	98.81912324	98.7663314	99.10741434	98.69569465	99.77240023
Binding Constraints	11, 21, 33	11, 21	11, 24, 33	11, 24	11, 24, 33	11	11, 21, 33

Notation: $Mean_i$ = the process mean of the dimension for i^{th} part

Table E-3 Data for an Original Condition of a Product for Verifying Effects of Quality Loss Coefficients

Information	Side of Part 1		Side of Part 2		Side of Part 3		Side of Envelope		Side of Gap	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Allowable Minimum Semi-Tolerance Zone (mm)	0.02	0.02	0.02	0.02	0.02	0.02	N/A	N/A	N/A	N/A
Allowable Maximum Semi-Tolerance Zone (mm)	0.085	0.085	0.085	0.085	0.085	0.085	N/A	N/A	N/A	N/A
Allowable Minimum Semi-Tolerance Zone Based on The Capability of the Machine (mm)	0.019	0.019	0.019	0.019	0.019	0.019	N/A	N/A	N/A	N/A
Specified Minimum C*pm Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone	2	2	2	2	2	2	N/A	N/A	N/A	N/A
Specified Minimum Proportion of Conformity (Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone)	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	2	2
Fixed Conversion Cost	280.7	280.7	280.7	280.7	280.7	280.7	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 1st Order	2407	2407	2407	2407	2407	2407	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 2nd Order	282.3	282.3	282.3	282.3	282.3	282.3	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 3rd Order	45960	45960	45960	45960	45960	45960	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 4th Order	106100	106100	106100	106100	106100	106100	N/A	N/A	N/A	N/A
Quality Loss Coefficient, K	13560	13560	10380	10380	12320	12320	N/A	N/A	N/A	N/A
Optimum Semi-Tolerance Zone (mm)	0.04	0.041	0.038	0.038	0.036	0.03	0.075	0.075	0.16	0.16

Table E-3 Data for an Original Condition of a Product for Verifying Effects of Quality Loss Coefficients

Information	Part 1	Part 2	Part 3	Envelope	Gap
Nominal Size (mm)	50.455	40.725	38.75	130.1	0.17
Process Mean of the Dimension (mm)	50.455	40.725	38.75	130.1	0.17
Minimum allowable Process Standard Deviation of the Dimension (mm)	0.012	0.012	0.012	N/A	N/A
Maximum allowable Process Standard Deviation of the Dimension (mm)	0.0156	0.0156	0.0156	N/A	0.026
Optimum Process Standard Deviation of the Dimension (mm)	0.013173	0.013036	0.012764	0.013	0.026
Mean Offsets from the Nominal Size (Numbers of Standard Deviation)	0	0	0	0	0
Inspection Strategy	NI	IWR	IIR	NI	NI
Multiplier for Determining Raw Conversion Cost	25	20	19	N/A	N/A
Inspection Cost (Measured as Percentage of Conversion Cost)	10	10	10	N/A	N/A
Scrap Cost (Measured as Percentage of Conversion Cost)	200	200	200	N/A	N/A
Reworking Cost (Measured as Percentage of Conversion Cost)	25	25	25	N/A	N/A

Optimum Total Cost ($) = 154.97508

Binding Constraint: 11

N/A : No Available Information

Table B-4 Model Verification Results: Effects of Quality Loss Coeffecients on Semi-Tolerance Zones, Costs and Binding Constraints

Condition Changed / Values from Optimal Solution	Value of Quality Loss Coefficient							
	Balance Quality Loss Coefficient NI, IWR and IIR for Part1, 2 and 3 (Original Condition)	K_{L1} increases to 5 times	K_{L1} increases to 25 times	K_{U1} increases to 5 times	K_{L2} increases to 5 times	K_{L2} increases to 10 times	K_{U2} increases to 5 times	K_{U2} increases to 10 times
Lower Semi-Tolerance Zone for Part 1 (mm)	0.04	0.039	0.031	0.039	0.041	0.042	0.041	0.042
Upper Semi-Tolerance Zone for Part 1 (mm)	0.041	0.039	0.031	0.039	0.041	0.043	0.041	0.043
Lower Semi-Tolerance Zone for Part 2 (mm)	0.038	0.038	0.042	0.038	0.035	0.031	0.038	0.038
Upper Semi-Tolerance Zone for Part 2 (mm)	0.038	0.039	0.042	0.039	0.038	0.038	0.035	0.031
Lower Semi-Tolerance Zone for Part 3 (mm)	0.036	0.037	0.04	0.037	0.037	0.037	0.037	0.037
Upper Semi-Tolerance Zone for Part 3 (mm)	0.03	0.031	0.037	0.031	0.031	0.032	0.031	0.032
Conversion Cost for Lower Side of Part 1 ($)	26.13450541	26.5693282	30.24211834	26.5693282	25.72100581	25.3032101	25.72100581	25.3032101
Conversion Cost for Upper Side of Part 1 ($)	25.72793512	26.56932822	30.24211837	26.56932822	25.72100584	24.90639644	25.72100584	24.90639644
Conversion Cost for Lower Side of Part 2 ($)	21.60341748	21.59479643	20.24621694	21.59479643	22.64061113	24.03842207	21.64189591	21.74206595
Conversion Cost for Upper Side of Part 2 ($)	21.6034175	21.26394477	20.24621696	21.26394477	21.64189593	21.74206597	22.64061115	24.03842209
Conversion Cost for Lower Side of Part 3 ($)	21.35015752	20.98232843	19.89172314	20.98232843	20.98232843	20.95133119	20.98232843	20.95133119
Conversion Cost for Upper Side of Part 3 ($)	23.19068642	22.84832519	20.83383559	22.84832519	22.84832519	22.51734588	22.84832519	22.51734588
Scrap Cost for Part1 ($)	0	0	0	0	0	0	0	0
Scrap Cost for Part2 ($)	0.015372537	0.01384386	0.00620031	0.01384386	0.022697311	0.043268684	0.022697311	0.043268684
Scrap Cost for Part3 ($)	0.011346665	0.009056682	0.004724908	0.009056682	0.009056682	0.009145206	0.009056682	0.009145206
Reworking Cost for Part1 ($)	0	0	0	0	0	0	0	0
Reworking Cost for Part2 ($)	0	0	0	0	0	0	0	0
Reworking Cost for Part3 ($)	0.005546891	0.00453003	0.001241173	0.00453003	0.00453003	0.003664671	0.00453003	0.003664671
Expected Quality Loss for Lower Side of Part 1 ($)	1.176470642	5.809507435	27.14330974	1.161901487	1.181347199	1.196037386	1.181347199	1.196037386
Expected Quality Loss for Upper Side of Part 1 ($)	1.176470643	1.161901488	1.085732391	5.809507441	1.181347201	1.196037387	1.181347201	1.196037387
Expected Quality Loss for Lower Side of Part 2 ($)	0.852610485	0.855372748	0.896552285	0.855372748	4.10185581	7.603793294	0.844797972	0.836203556
Expected Quality Loss for Upper Side of Part 2 ($)	0.852610486	0.861506962	0.896552286	0.861506962	0.844797973	0.836203557	4.101855814	7.603793302
Expected Quality Loss for Lower Side of Part 3 ($)	1.003528145	1.012123636	1.051256945	1.012123636	1.012123636	1.016435127	1.012123636	1.016435127
Expected Quality Loss for Upper Side of Part 3 ($)	0.882023772	0.885718109	0.911794642	0.885718109	0.870987087	0.856379596	4.354935436	8.563795962
Total Cost for Part1 ($)	54.21538182	60.11006534	88.71327884	60.11006535	53.80470605	52.60168131	53.80470605	52.60168131
Total Cost for Part2 ($)	49.54019018	49.13837223	46.45878807	49.13837223	54.11135777	59.66390738	54.11135778	59.66390739
Total Cost for Part3 ($)	51.21950672	50.40290341	46.9612168	50.40290341	50.40290341	50.00314473	50.40290341	50.00314473
Grand Total Cost ($)	154.9750787	159.651341	182.1332837	159.651341	158.3189672	162.2687334	158.3189672	162.2687334
Binding Constraint	11	11	11	11	11	11	11	11

Table E-4 Model Verification Results: Effects of Quality Loss Coeffecients on Semi-Tolerance Zones, Costs and Binding Constraints (Continued)

Condition Changed / Values from Optimal Solution	Value of Quality Loss Coefficient							
	K_{L3} increases to 5 times	K_{U3} increases to 5 times	K_{L1} and K_{L2} increase to 5 times	K_{L1} and K_{U2} increase to 10 times	K_{U1} and K_{L2} increase to 10 times	K_{U1} and K_{U2} increase to 10 times	K_{L1} and K_{L3} increase to 10 times	K_{L1} and K_{U3} increase to 10 times
Lower Semi-Tolerance Zone for Part 1 (mm)	0.041	0.042	0.04	0.039	0.039	0.039	0.039	0.039
Upper Semi-Tolerance Zone for Part 1 (mm)	0.042	0.042	0.04	0.039	0.039	0.039	0.039	0.04
Lower Semi-Tolerance Zone for Part 2 (mm)	0.038	0.038	0.036	0.039	0.032	0.039	0.041	0.041
Upper Semi-Tolerance Zone for Part 2 (mm)	0.038	0.039	0.038	0.032	0.039	0.032	0.041	0.041
Lower Semi-Tolerance Zone for Part 3 (mm)	0.033	0.036	0.037	0.039	0.039	0.039	0.028	0.036
Upper Semi-Tolerance Zone for Part 3 (mm)	0.031	0.026	0.032	0.035	0.035	0.035	0.035	0.026
Conversion Cost for Lower Side of Part 1 ($)	25.715332	25.30777118	26.14154799	26.5693282	26.5693282	26.5693282	26.5693282	26.56060639
Conversion Cost for Upper Side of Part 1 ($)	25.31335386	25.3077712	26.14154801	26.56932822	26.56932822	26.56932822	26.56932822	26.15012939
Conversion Cost for Lower Side of Part 2 ($)	21.60341748	21.59479643	22.29236731	21.36915922	23.68028924	21.36915922	20.57680465	20.57680465
Conversion Cost for Upper Side of Part 2 ($)	21.6034175	21.26394477	21.62654748	23.68028926	21.36915924	23.68028926	20.57680467	20.57680467
Conversion Cost for Lower Side of Part 3 ($)	22.31517998	21.58501043	20.95133119	20.23685545	20.23685545	20.23685545	23.84981749	21.58501043
Conversion Cost for Upper Side of Part 3 ($)	22.92231487	24.43719758	22.51734588	21.49983035	21.49983035	21.49983035	21.78214314	24.43719758
Scrap Cost for Part1 ($)	0	0	0	0	0	0	0	0
Scrap Cost for Part2 ($)	0.015372537	0.01384386	0.019704678	0.035175222	0.035175222	0.035175222	0.007803194	0.007803194
Scrap Cost for Part3 ($)	0.022585294	0.010982767	0.009145206	0.005870156	0.005870156	0.005870156	0.065640856	0.010982767
Reworking Cost for Part1 ($)	0	0	0	0	0	0	0	0
Reworking Cost for Part2 ($)	0	0	0	0	0	0	0	0
Reworking Cost for Part3 ($)	0.004413337	0.012332341	0.003664671	0.001933318	0.001933318	0.001933318	0.001741086	0.012332341
Expected Quality Loss for Lower Side of Part 1 ($)	1.186233842	1.191130571	5.858020857	11.61901487	1.161901487	1.161901487	11.61901487	11.66747786
Expected Quality Loss for Upper Side of Part 1 ($)	1.186233843	1.191130572	1.171604172	1.161901488	11.61901488	11.61901488	1.161901488	1.166747787
Expected Quality Loss for Lower Side of Part 2 ($)	0.852610485	0.855372748	4.160746186	0.846539481	7.796077624	0.846539481	0.886261425	0.886261425
Expected Quality Loss for Upper Side of Part 2 ($)	0.852610486	0.861506962	0.847307499	7.796077632	0.846539482	7.796077632	0.886261426	0.886261426
Expected Quality Loss for Lower Side of Part 3 ($)	4.974846543	0.986447127	1.016435127	1.038130036	1.038130036	1.038130036	9.907036359	0.986447127
Expected Quality Loss for Upper Side of Part 3 ($)	0.882023772	0.885718109	0.874658262	8.636679004	0.8636679	8.636679004	0.9043056	0.9043056
Total Cost for Part1 ($)	53.40115354	52.99780352	59.31272103	65.91957278	65.91957279	65.91957279	65.91957278	65.54496143
Total Cost for Part2 ($)	49.54019018	49.13837223	53.71295351	58.90051489	58.90051488	58.90051489	47.19755699	47.19755699
Total Cost for Part3 ($)	55.87074401	56.06424949	50.00314473	48.06990695	48.06990695	48.06990695	60.71574355	59.98341222
Grand Total Cost ($)	158.8120877	158.2004252	163.0288193	172.8899946	172.8899946	172.8899946	173.8328733	172.7259306
Binding Constraint	11	11	11	11	11	11	11	11

Table E-4 Model Verification Results: Effects of Quality Loss Coeffecients on Semi-Tolerance Zones, Costs and Binding Constraints (Continued)

Condition Changed / Values from Optimal Solution	Value of Quality Loss Coefficient							
	K_{U1} and K_{L3} increase to 10 times	K_{U1} and K_{U3} increase to 10 times	K_{L2} and K_{L3} increase to 10 times	K_{L2} and K_{U3} increase to 10 times	K_{U2} and K_{L3} increase to 10 times	K_{U2} and K_{U3} increase to 10 times	K_{L1}, K_{L2} and K_{L3} increase to 10 times	K_{U1}, K_{U2} and K_{U3} increase to 10 times
Lower Semi-Tolerance Zone for Part 1 (mm)	0.039	0.039	0.045	0.045	0.045	0.045	0.042	0.043
Upper Semi-Tolerance Zone for Part 1 (mm)	0.039	0.04	0.045	0.045	0.045	0.045	0.042	0.043
Lower Semi-Tolerance Zone for Part 2 (mm)	0.041	0.041	0.032	0.032	0.039	0.039	0.033	0.041
Upper Semi-Tolerance Zone for Part 2 (mm)	0.041	0.041	0.039	0.039	0.032	0.032	0.04	0.034
Lower Semi-Tolerance Zone for Part 3 (mm)	0.028	0.036	0.028	0.036	0.028	0.036	0.029	0.036
Upper Semi-Tolerance Zone for Part 3 (mm)	0.035	0.026	0.034	0.026	0.034	0.026	0.037	0.026
Conversion Cost for Lower Side of Part 1 ($)	26.5693282	26.56060639	24.11253111	24.11253111	24.11253111	24.11253111	25.30777118	24.90190849
Conversion Cost for Upper Side of Part 1 ($)	26.56932822	26.15012939	24.11253114	24.11253114	24.11253114	24.11253114	25.3077712	24.90190851
Conversion Cost for Lower Side of Part 2 ($)	20.57680465	20.57680465	23.68028924	23.68028924	21.36915922	21.36915922	23.3226002	20.65269127
Conversion Cost for Upper Side of Part 2 ($)	20.57680467	20.57680467	21.36915924	21.36915924	23.68028926	23.68028926	21.00623998	22.96619646
Conversion Cost for Lower Side of Part 3 ($)	23.84981749	21.58501043	23.87127958	21.58501043	23.87127958	21.58501043	23.49941729	21.58501043
Conversion Cost for Upper Side of Part 3 ($)	21.78214314	24.43719758	22.11818037	24.43719758	22.11818037	24.43719758	21.0635988	24.43719758
Scrap Cost for Part1 ($)	0	0	0	0	0	0	0	0
Scrap Cost for Part2 ($)	0.007803194	0.007803194	0.035175222	0.035175222	0.035175222	0.035175222	0.028527939	0.023083705
Scrap Cost for Part3 ($)	0.065640856	0.010982767	0.065400873	0.010982767	0.065400873	0.010982767	0.054237714	0.010982767
Reworking Cost for Part1 ($)	0	0	0	0	0	0	0	0
Reworking Cost for Part2 ($)	0	0	0	0	0	0	0	0
Reworking Cost for Part3 ($)	0.001741086	0.012332341	0.002190739	0.012332341	0.002190739	0.012332341	0.001100113	0.012332341
Expected Quality Loss for Lower Side of Part 1 ($)	1.161901487	1.166747786	1.22072275	1.22072275	1.22072275	1.22072275	11.91130571	1.200954287
Expected Quality Loss for Upper Side of Part 1 ($)	11.61901488	11.66747787	1.220722751	1.220722751	1.220722751	1.220722751	1.191130572	12.00954288
Expected Quality Loss for Lower Side of Part 2 ($)	0.886261425	0.886261425	7.796077624	7.796077624	0.846539481	0.846539481	7.97649694	0.866104238
Expected Quality Loss for Upper Side of Part 2 ($)	0.886261426	0.886261426	0.846539482	0.846539482	7.796077632	7.796077632	0.85649037	8.145812891
Expected Quality Loss for Lower Side of Part 3 ($)	9.907036359	0.986447127	9.864471268	0.986447127	9.864471268	0.986447127	10.03528145	0.986447127
Expected Quality Loss for Upper Side of Part 3 ($)	0.9043056	0.9043056	0.8636679	0.8636679	8.636679004	8.636679004	0.870987087	8.783371566
Total Cost for Part1 ($)	65.91957279	65.54496144	50.66650775	50.66650775	50.66650775	50.66650775	63.71797866	63.01431416
Total Cost for Part2 ($)	47.19755699	47.19755699	58.90051488	58.90051488	58.90051489	58.90051489	58.16527028	57.45436773
Total Cost for Part3 ($)	60.71574355	59.98341222	61.09405706	59.98341222	61.09405706	59.98341222	59.61641488	59.98341222
Grand Total Cost ($)	173.8328733	172.7259307	170.6610797	169.5504349	170.6610797	169.5504349	181.4996638	180.4520941
Binding Constraint	11	11	11	11	11	11	11	11

Table E-5 Data for an Original Condition of a Product for Verifying Effects of Constraints

Information	Side of Part 1		Side of Part 2		Side of Part 3		Side of Envelope		Side of Gap	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Allowable Minimum Semi-Tolerance Zone (mm)	0.019	0.019	0.019	0.019	0.019	0.019	N/A	N/A	N/A	N/A
Allowable Maximum Semi-Tolerance Zone (mm)	0.085	0.085	0.085	0.085	0.085	0.085	N/A	N/A	N/A	N/A
Allowable Minimum Semi-Tolerance Zone Based on The Capability of the Machine (mm)	0.019	0.019	0.019	0.019	0.019	0.019	N/A	N/A	N/A	N/A
Specified Minimum C*pm Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone	4	4	4	4	4	4	N/A	N/A	N/A	N/A
Specified Minimum Proportion of Conformity (Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone)	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	3	3
Fixed Conversion Cost	280.7	280.7	280.7	280.7	280.7	280.7	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 1st Order	2407	2407	2407	2407	2407	2407	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 2nd Order	282.3	282.3	282.3	282.3	282.3	282.3	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 3rd Order	45960	45960	45960	45960	45960	45960	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 4th Order	106100	106100	106100	106100	106100	106100	N/A	N/A	N/A	N/A
Quality Loss Coefficient, K	20340	13560	15570	10380	12320	18480	N/A	N/A	N/A	N/A
Optimum Semi-Tolerance Zone (mm)	0.078	0.085	0.073	0.085	0.085	0.069	0.075	0.075	0.16	0.16
Number of Standard Deviation in the Optimum Semi-tolerance Zone	5.0619469	5.5162242	4.7797619	5.5654762	5.6055156	4.5503597	5.7692308	5.7692308	5.4237288	5.4237288

Table E-5 Data for an Original Condition of a Product for Verifying Effects of Constraints (Continued)

Information	Part 1	Part 2	Part 3	Envelope	Gap
Nominal Size (mm)	50.455	40.725	38.75	130.1	0.17
Process Mean of the Dimension (mm)	50.459	40.729	38.746	130.106	0.172
Minimum allowable Process Standard Deviation of the Dimension (mm)	0.012	0.012	0.012	N/A	N/A
Maximum allowable Process Standard Deviation of the Dimension (mm)	0.0156	0.0156	0.0156	N/A	0.0295
Optimum Process Standard Deviation of the Dimension (mm)	0.0154091	0.0152727	0.0151636	0.013	0.0295
Mean Offsets from the Nominal Size (Numbers of Standard Deviation)	0.259587	0.2619048	0.263789	0.4615385	0.0677966
Inspection Strategy	IIR	IWR	NI	NI	NI
Multiplier for Determining Raw Conversion Cost	25	20	19	N/A	N/A
Inspection Cost (Measured as Percentage of Conversion Cost)	10	10	10	N/A	N/A
Scrap Cost (Measured as Percentage of Conversion Cost)	200	200	200	N/A	N/A
Reworking Cost (Measured as Percentage of Conversion Cost)	25	25	25	N/A	N/A

Optimum Total Cost ($) = 93.965244

Binding Constraint: 11, 23, 24, 25

N/A : No Available Information

Table B-6 Model Verification Results: Effects of Constraints on Semi-Tolerance Zones, Costs and Binding Constraints

Condition Changed / Values from Optimal Solution	Value of Constraint: (Original Condition)	$Sigma_{max C}$ Changes from .0295 to .029 mm.	$Sigma_{max C}$ Changes from .0295 to .03 mm.	$Delta_{max L1}$ Changes from .085 to .075 mm.	$Delta_{max L1}$ Changes from .085 to .09 mm.	$Delta_{max L2}$ Changes from .085 to .075 mm.	$Delta_{max L2}$ Changes from .085 to .09 mm.	$Delta_{max L3}$ Changes from .085 to .075 mm.	$Delta_{max L3}$ Changes from .085 to .09 mm.
Lower Semi-Tolerance Zone for Part 1 (mm)	0.078	0.07	0.085	0.071	0.079	0.075	0.079	0.079	0.078
Upper Semi-Tolerance Zone for Part 1 (mm)	0.085	0.085	0.085	0.085	0.085	0.085	0.085	0.085	0.085
Lower Semi-Tolerance Zone for Part 2 (mm)	0.073	0.064	0.085	0.072	0.075	0.07	0.075	0.074	0.074
Upper Semi-Tolerance Zone for Part 2 (mm)	0.085	0.083	0.085	0.085	0.085	0.085	0.085	0.085	0.085
Lower Semi-Tolerance Zone for Part 3 (mm)	0.085	0.079	0.085	0.085	0.085	0.085	0.085	0.075	0.088
Upper Semi-Tolerance Zone for Part 3 (mm)	0.069	0.059	0.085	0.068	0.071	0.066	0.071	0.069	0.07
Conversion Cost for Lower Side of Part 1 ($)	11.87997989	12.74970979	11.35533728	12.67864437	11.76821039	12.16981598	11.79247908	11.79247908	11.87997989
Conversion Cost for Upper Side of Part 1 ($)	18.14738269	18.190694	18.11045287	18.1331924	18.16671288	18.16348285	18.14205283	18.14205283	18.14738269
Conversion Cost for Lower Side of Part 2 ($)	9.909443101	10.88375609	9.084269824	10.00216013	9.735852782	10.24400907	9.716153943	9.820697123	9.820697123
Conversion Cost for Upper Side of Part 2 ($)	14.53944784	14.84261741	14.48836229	14.54380145	14.53078628	14.51114444	14.55014747	14.5351095	14.5351095
Conversion Cost for Lower Side of Part 3 ($)	13.82910799	14.69449378	13.76394418	13.83330389	13.82076198	13.84174317	13.82076198	15.27631987	13.5092692
Conversion Cost for Upper Side of Part 3 ($)	9.789541125	10.98237891	8.630056333	9.893306408	9.593968485	10.11304273	9.593968485	9.783613843	9.683617817
Scrap Cost for Part1 ($)	1.2385E-07	1.37377E-06	1.37419E-08	1.56772E-06	7.11466E-08	3.09602E-07	9.09317E-08	9.09317E-08	1.2385E-07
Scrap Cost for Part2 ($)	7.0419E-07	7.35858E-06	2.59012E-07	8.93859E-07	4.6269E-07	2.27566E-06	3.69836E-07	5.646E-07	5.646E-07
Scrap Cost for Part3 ($)	0	0	0	0	0	0	0	0	0
Reworking Cost for Part1 ($)	2.20713E-08	1.50482E-08	3.06587E-08	2.58992E-08	1.83123E-08	1.9136E-08	2.31409E-08	2.31409E-08	2.20713E-08
Reworking Cost for Part2 ($)	0	0	0	0	0	0	0	0	0
Reworking Cost for Part3 ($)	0	0	0	0	0	0	0	0	0
Expected Quality Loss for Lower Side of Part 1 ($)	1.56598805	1.512040686	1.613942649	1.584141538	1.541628666	1.545655752	1.572794343	1.572794343	1.56598805
Expected Quality Loss for Upper Side of Part 1 ($)	2.392643952	2.338046024	2.44094911	2.411026607	2.368004134	2.372093581	2.399514335	2.399514335	2.392643952
Expected Quality Loss for Lower Side of Part 2 ($)	1.172856374	1.116781596	1.235451545	1.167710412	1.183178886	1.206967818	1.160282448	1.178012425	1.178012425
Expected Quality Loss for Upper Side of Part 2 ($)	1.805357506	1.748445567	1.868513948	1.800144909	1.815806034	1.83990513	1.792586546	1.810577884	1.810577884
Expected Quality Loss for Lower Side of Part 3 ($)	2.118085986	2.020784213	2.217744877	2.111935559	2.130414462	2.099662329	2.130414462	2.110021733	2.115750433
Expected Quality Loss for Upper Side of Part 3 ($)	1.805361524	1.748445592	1.868522064	1.80014856	1.815810721	1.839909799	1.792590262	1.810582244	1.810582244
Total Cost for Part1 ($)	36.98873471	37.88456575	36.46726234	37.88822869	36.83805082	37.28438645	36.90029686	36.90029686	36.98873471
Total Cost for Part2 ($)	29.872008	31.16438518	29.033866	29.96843094	29.69229714	30.27758732	29.64580794	29.77998888	29.77998888
Total Cost for Part3 ($)	27.10450096	28.97033513	26.07810087	27.20026616	26.92504294	27.40411844	26.92504294	28.52979549	26.67410703
Grand Total Cost ($)	93.96524367	98.01928606	91.57922922	95.05692579	93.4553909	94.96609222	93.47114774	95.21008123	93.44283062
Binding Constraints	11, 23, 24, 25	11, 24, 36	21, 22, 23, 24, 25, 26, 41, 42, 43	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25	11, 24, 25

Notation for Table E-6: Model Verification Results: Effects of Constraint Changes on Semi-Tolerance Zones, Costs and Binding Constraints:

$Sigma_{maxG}$ = Maximum Allowable Process Standard Deviation for Producing the Gap

$Sigma_{maxi}$ = Maximum Allowable Process Standard Deviation for Producing i^{th} Part

$Delta_{max\ Li}$ = Allowable Maximum Value for Lower Semi-tolerance Zone for i^{th} Part

$Delta_{max\ Ui}$ = Allowable Maximum Value for Upper Semi-tolerance Zone for i^{th} Part

Table E-6 Model Verification Results: Effects of Constraints on Semi-Tolerance Zones, Costs and Binding Constraints (Continued)

Condition Changed / Values from Optimal Solution	Value of Constraint									
	$Delta_{max\ u1}$ Changes from .085 to .08 mm.	$Delta_{max\ u1}$ Changes from .085 to .09 mm.	$Delta_{max\ u2}$ Changes from .085 to .08 mm.	$Delta_{max\ u2}$ Changes from .085 to .09 mm.	$Delta_{max\ u3}$ Changes from .085 to .08 mm.	$Delta_{max\ u3}$ Changes from .085 to .09 mm.	L_1 Changes from 4 to 2.9	L_1 Changes from 4 to 5.1	L_2 Changes from 4 to 3	L_2 Changes from 4 to 5
Lower Semi-Tolerance Zone for Part 1 (mm)	0.078	0.078	0.078	0.078	0.076	0.079	0.078	0.079	0.078	0.076
Upper Semi-Tolerance Zone for Part 1 (mm)	0.08	0.09	0.085	0.085	0.085	0.085	0.085	0.085	0.085	0.085
Lower Semi-Tolerance Zone for Part 2 (mm)	0.074	0.073	0.074	0.073	0.072	0.075	0.073	0.073	0.073	0.077
Upper Semi-Tolerance Zone for Part 2 (mm)	0.085	0.085	0.08	0.09	0.085	0.085	0.085	0.085	0.085	0.085
Lower Semi-Tolerance Zone for Part 3 (mm)	0.085	0.085	0.085	0.085	0.085	0.085	0.085	0.085	0.085	0.085
Upper Semi-Tolerance Zone for Part 3 (mm)	0.069	0.069	0.069	0.069	0.068	0.071	0.069	0.068	0.069	0.067
Conversion Cost for Lower Side of Part 1 ($)	11.87852379	11.88133137	11.87997989	11.87997989	12.06854654	11.79247908	11.87997989	11.79247908	11.87997989	12.06854654
Conversion Cost for Upper Side of Part 1 ($)	18.99628709	17.47645293	18.14738269	18.14738269	18.15809758	18.14205283	18.14738269	18.14205283	18.14738269	18.15809758
Conversion Cost for Lower Side of Part 2 ($)	9.820697123	9.909443101	9.818773034	9.911413449	10.00216013	9.735852782	9.909443101	9.909443101	9.909443101	9.577574182
Conversion Cost for Upper Side of Part 2 ($)	14.5351095	14.53944784	15.21576846	14.0011282	14.54380145	14.53078628	14.53944784	14.53944784	14.53944784	14.52218472
Conversion Cost for Lower Side of Part 3 ($)	13.82910799	13.82910799	13.82910799	13.82910799	13.81444039	13.83879482	13.82910799	13.83330389	13.82910799	13.83751548
Conversion Cost for Upper Side of Part 3 ($)	9.789541125	9.789541125	9.789541125	9.789541125	9.91390157	9.574902328	9.789541125	9.893306408	9.789541125	10.00112486
Scrap Cost for Part1 ($)	1.25541E-07	1.227E-07	1.2385E-07	1.2385E-07	2.28534E-07	9.09317E-08	1.2385E-07	9.09317E-08	1.2385E-07	2.28534E-07
Scrap Cost for Part2 ($)	5.646E-07	7.0419E-07	1.25583E-06	5.86217E-07	8.93859E-07	4.6269E-07	7.0419E-07	7.0419E-07	7.0419E-07	3.36777E-07
Scrap Cost for Part3 ($)	0	0	0	0	0	0	0	0	0	0
Reworking Cost for Part1 ($)	1.24195E-07	3.56845E-09	2.20713E-08	2.20713E-08	2.00707E-08	2.31409E-08	2.20713E-08	2.31409E-08	2.20713E-08	2.00707E-08
Reworking Cost for Part2 ($)	0	0	0	0	0	0	0	0	0	0
Reworking Cost for Part3 ($)	0	0	0	0	0	0	0	0	0	0
Expected Quality Loss for Lower Side of Part 1 ($)	1.564116531	1.567724743	1.56598805	1.56598805	1.552418933	1.572794343	1.56598805	1.572794343	1.56598805	1.552418933
Expected Quality Loss for Upper Side of Part 1 ($)	2.390723963	2.394404265	2.392643952	2.392643952	2.37893357	2.399514335	2.392643952	2.399514335	2.392643952	2.37893357
Expected Quality Loss for Lower Side of Part 2 ($)	1.178012425	1.172856374	1.175780814	1.175110478	1.167710412	1.183178886	1.172856374	1.172856374	1.172856374	1.193543986
Expected Quality Loss for Upper Side of Part 2 ($)	1.810577884	1.805357506	1.80829751	1.807646641	1.800144909	1.815806034	1.805357506	1.805357506	1.805357506	1.826285622
Expected Quality Loss for Lower Side of Part 3 ($)	2.118085986	2.118085986	2.118085986	2.118085986	2.139855506	2.103913474	2.118085986	2.111935559	2.118085986	2.10579434
Expected Quality Loss for Upper Side of Part 3 ($)	1.810582244	1.805361524	1.80832035	1.807647016	1.80014856	1.815810721	1.805361524	1.805361524	1.805361524	1.826290944
Total Cost for Part1 ($)	37.91713995	36.25569494	36.98873471	36.98873471	37.18066744	36.90029686	36.98873471	36.90029686	36.98873471	37.18066744
Total Cost for Part2 ($)	29.77998888	29.872008	30.52209908	29.28656466	29.96843094	29.69229714	29.872008	29.872008	29.872008	29.52957114
Total Cost for Part3 ($)	27.10450096	27.10450096	27.10450096	27.10450096	27.25739864	26.87145259	27.10450096	27.20026616	27.10450096	27.3001231
Grand Total Cost ($)	94.80162978	93.23220389	94.61533475	93.37980033	94.40649702	93.46404659	93.96524367	93.97257101	93.96524367	94.01036167
Binding Constraints	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25, 31	11, 23, 24, 25	11, 23, 24,25 32

Table E-6 Model Verification Result: Effects of Constraints on Semi-Tolerance Zones, Costs and Binding Constraints (Continued)

Condition Changed / Values from Optimal Solution	Value of Constraint									
	L_{L3} Changes from 4 to 3	L_{L3} Changes from 4 to 5	L_{U1} Changes from 4 to 2	L_{U1} Changes from 4 to 6	L_{U2} Changes from 4 to 3	L_{U2} Changes from 4 to 5	L_{U3} Changes from 4 to 3	L_{U3} Changes from 4 to 5	$Sigma_{max1}$ Changes from .0156 to .0151 mm.	$Sigma_{max1}$ Changes from .0156 to .0161 mm.
Lower Semi-Tolerance Zone for Part 1 (mm)	0.078	0.078	0.078	Infeasible	0.078	0.078	0.078	0.074	0.083	0.071
Upper Semi-Tolerance Zone for Part 1 (mm)	0.085	0.085	0.085	Solution	0.085	0.085	0.085	0.085	0.085	0.085
Lower Semi-Tolerance Zone for Part 2 (mm)	0.073	0.073	0.073		0.073	0.073	0.073	0.07	0.079	0.069
Upper Semi-Tolerance Zone for Part 2 (mm)	0.085	0.085	0.085		0.085	0.085	0.085	0.085	0.085	0.085
Lower Semi-Tolerance Zone for Part 3 (mm)	0.085	0.085	0.085		0.085	0.085	0.085	0.084	0.085	0.084
Upper Semi-Tolerance Zone for Part 3 (mm)	0.069	0.069	0.069		0.069	0.069	0.069	0.077	0.076	0.065
Conversion Cost for Lower Side of Part 1 ($)	11.87997989	11.87997989	11.87997989		11.87997989	11.87997989	11.87997989	12.2758714	11.39158173	12.71578517
Conversion Cost for Upper Side of Part 1 ($)	18.14738269	18.14738269	18.14738269		18.14738269	18.14738269	18.14738269	18.16888687	18.21867315	18.09806451
Conversion Cost for Lower Side of Part 2 ($)	9.909443101	9.909443101	9.909443101		9.909443101	9.909443101	9.909443101	10.19976783	9.433983264	10.30478013
Conversion Cost for Upper Side of Part 2 ($)	14.53944784	14.53944784	14.53944784		14.53944784	14.53944784	14.53944784	14.5525552	14.51364226	14.55695578
Conversion Cost for Lower Side of Part 3 ($)	13.82910799	13.82910799	13.82910799		13.82910799	13.82910799	13.82910799	13.91771934	13.80015416	13.96822998
Conversion Cost for Upper Side of Part 3 ($)	9.789541125	9.789541125	9.789541125		9.789541125	9.789541125	9.789541125	9.094617443	9.172095369	10.22429856
Scrap Cost for Part1 ($)	1.2385E-07	1.2385E-07	1.2385E-07		1.2385E-07	1.2385E-07	1.2385E-07	4.18659E-07	8.89757E-09	2.08204E-06
Scrap Cost for Part2 ($)	7.0419E-07	7.0419E-07	7.0419E-07		7.0419E-07	7.0419E-07	7.0419E-07	1.49415E-06	2.76059E-07	1.95474E-06
Scrap Cost for Part3 ($)	0	0	0		0	0	0	0	0	0
Reworking Cost for Part1 ($)	2.20713E-08	2.20713E-08	2.20713E-08		2.20713E-08	2.20713E-08	2.20713E-08	1.82426E-08	1.09926E-08	3.59961E-08
Reworking Cost for Part2 ($)	0	0	0		0	0	0	0	0	0
Reworking Cost for Part3 ($)	0	0	0		0	0	0	0	0	0
Expected Quality Loss for Lower Side of Part 1 ($)	1.56598805	1.56598805	1.56598805		1.56598805	1.56598805	1.56598805	1.538906498	1.478537614	1.630383114
Expected Quality Loss for Upper Side of Part 1 ($)	2.392643952	2.392643952	2.392643952		2.392643952	2.392643952	2.392643952	2.365263736	2.303875143	2.457549678
Expected Quality Loss for Lower Side of Part 2 ($)	1.172856374	1.172856374	1.172856374		1.172856374	1.172856374	1.172856374	1.157447018	1.203953098	1.15232842
Expected Quality Loss for Upper Side of Part 2 ($)	1.805357506	1.805357506	1.805357506		1.805357506	1.805357506	1.805357506	1.789743108	1.836796229	1.78455394
Expected Quality Loss for Lower Side of Part 3 ($)	2.118085986	2.118085986	2.118085986		2.118085986	2.118085986	2.118085986	2.161396771	2.161396771	2.087425931
Expected Quality Loss for Upper Side of Part 3 ($)	1.805361524	1.805361524	1.805361524		1.805361524	1.805361524	1.805361524	1.789745904	1.836802191	1.784556212
Total Cost for Part1 ($)	36.98873471	36.98873471	36.98873471		36.98873471	36.98873471	36.98873471	37.39341544	36.35369373	37.98322074
Total Cost for Part2 ($)	29.872008	29.872008	29.872008		29.872008	29.872008	29.872008	30.17477534	29.38314293	30.28483095
Total Cost for Part3 ($)	27.10450096	27.10450096	27.10450096		27.10450096	27.10450096	27.10450096	26.58420138	26.54411412	27.61762932
Grand Total Cost ($)	93.96524367	93.96524367	93.96524367		93.96524367	93.96524367	93.96524367	94.15239216	92.28095078	95.88568101
Binding Constraints	11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25		11, 23, 24, 25	11, 23, 24, 25	11, 23, 24, 25	11, 24, 25, 36	11, 23, 24, 25	11, 24, 25

Table E-6 Model Verification Result: Effects of Constraints on Semi-Tolerance Zones, Costs and Binding Constraints (Continued)

Condition Changed / Values from Optimal Solution	Value of constraint			
	$Sigma_{max2}$ Changes from .0156 to .0151 mm.	$Sigma_{max2}$ Changes from .0156 to .0161 mm.	$Sigma_{max3}$ Changes from .0156 to .0151 mm.	$Sigma_{max3}$ Changes from .0156 to .0161 mm.
Lower Semi-Tolerance Zone for Part 1 (mm)	0.083	0.075	0.082	0.074
Upper Semi-Tolerance Zone for Part 1 (mm)	0.085	0.085	0.085	0.085
Lower Semi-Tolerance Zone for Part 2 (mm)	0.079	0.066	0.079	0.069
Upper Semi-Tolerance Zone for Part 2 (mm)	0.085	0.084	0.085	0.085
Lower Semi-Tolerance Zone for Part 3 (mm)	0.085	0.084	0.085	0.083
Upper Semi-Tolerance Zone for Part 3 (mm)	0.076	0.066	0.076	0.064
Conversion Cost for Lower Side of Part 1 ($)	11.48511623	12.16981598	11.55579465	12.2758714
Conversion Cost for Upper Side of Part 1 ($)	18.12091463	18.16348285	18.1261722	18.16888687
Conversion Cost for Lower Side of Part 2 ($)	9.358421719	10.71587885	9.433983264	10.30478013
Conversion Cost for Upper Side of Part 2 ($)	14.59042232	14.63067051	14.51364226	14.55695578
Conversion Cost for Lower Side of Part 3 ($)	13.80015416	13.96393484	13.87205107	14.03616771
Conversion Cost for Upper Side of Part 3 ($)	9.172095369	10.10831134	9.09958595	10.41190466
Scrap Cost for Part1 ($)	2.59723E-08	3.09602E-07	3.56176E-08	4.18659E-07
Scrap Cost for Part2 ($)	1.08251E-07	8.05892E-06	2.76059E-07	1.95474E-06
Scrap Cost for Part3 ($)	0	0	0	0
Reworking Cost for Part1 ($)	2.79286E-08	1.9136E-08	2.6651E-08	1.82426E-08
Reworking Cost for Part2 ($)	0	0	0	0
Reworking Cost for Part3 ($)	0	0	0	0
Expected Quality Loss for Lower Side of Part 1 ($)	1.600166979	1.545655752	1.593301552	1.538906498
Expected Quality Loss for Upper Side of Part 1 ($)	2.427097062	2.372093581	2.420186205	2.365263736
Expected Quality Loss for Lower Side of Part 2 ($)	1.114464764	1.211858183	1.203953098	1.15232842
Expected Quality Loss for Upper Side of Part 2 ($)	1.745878442	1.844995675	1.836796229	1.78455394
Expected Quality Loss for Lower Side of Part 3 ($)	2.161396771	2.093539525	2.056490878	2.168486034
Expected Quality Loss for Upper Side of Part 3 ($)	1.745880809	1.844999054	1.836802191	1.784556212
Total Cost for Part1 ($)	36.59389961	37.28438645	36.66365311	37.39341544
Total Cost for Part2 ($)	29.20407382	30.93821933	29.38314293	30.28483095
Total Cost for Part3 ($)	26.54411412	27.50945141	26.33556077	28.03403717
Grand Total Cost ($)	92.34208755	95.73205719	92.38235681	95.71228357
Binding Constraints	11, 23, 24, 25	11, 24	11, 23, 24, 25	11, 24, 25

Table E-7 Data for an Original Condition of a Product for Verifying Effects of Constraint 51 and 54

Information	Side of Part 1		Side of Part 2		Side of Part 3		Side of Envelope		Side of Gap	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Allowable Minimum Semi-Tolerance Zone (mm)	0.019	0.019	0.019	0.019	0.019	0.019	N/A	N/A	N/A	N/A
Allowable Maximum Semi-Tolerance Zone (mm)	0.085	0.085	0.085	0.085	0.085	0.085	N/A	N/A	N/A	N/A
Allowable Minimum Semi-Tolerance Zone Based on The Capability of the Machine (mm)	0.019	0.019	0.019	0.019	0.019	0.019	N/A	N/A	N/A	N/A
Specified Minimum C*pm Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone	2.5	2.5	2.5	2.5	2.5	2.5	N/A	N/A	N/A	N/A
Specified Minimum Proportion of Conforming Part (Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone)	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	2.5	2.5
Fixed Conversion Cost	280.7	280.7	280.7	280.7	280.7	280.7	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 1st Order	2407	2407	2407	2407	2407	2407	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 2nd Order	282.3	282.3	282.3	282.3	282.3	282.3	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 3rd Order	45960	45960	45960	45960	45960	45960	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 4th Order	106100	106100	106100	106100	106100	106100	N/A	N/A	N/A	N/A
Quality Loss Coefficient, K	915300	610200	700650	467100	554400	831600	N/A	N/A	N/A	N/A
Optimum Semi-Tolerance Zone (mm)	0.032	0.032	0.032	0.034	0.032	0.032	0.075	0.075	0.13	0.13
Number of Standard Deviation in the Optimum Semi-tolerance Zone	2.5178827	2.5178827	2.5071225	2.6638177	2.5178827	2.5178827	5.7692308	5.7692308	5.0368074	5.0368074

Table E-7 Data for an Original Condition of a Product for Verifying Effects of Constraint 51 and 54 (Continued)

Information	Part 1	Part 2	Part 3	Envelope	Gap
Nominal Size (mm)	50.455	40.725	38.75	130.1	0.17
Process Mean of the Dimension (mm)	50.459	40.729	38.746	130.106	0.172
Minimum allowable Process Standard Deviation of the Dimension (mm)	0.012	0.012	0.012	N/A	N/A
Maximum allowable Process Standard Deviation of the Dimension (mm)	0.0156	0.0156	0.0156	N/A	0.03
Optimum Process Standard Deviation of the Dimension (mm)	0.0127091	0.0127636	0.0127091	0.013	0.02581
Mean Offsets from the Nominal Size (Numbers of Standard Deviation)	0.3147353	0.3133903	0.3147353	0.4615385	0.0774893
Inspection Strategy	IIR	IWR	IIR	NI	NI
Multiplier for Determining Raw Conversion Cost	25	20	19	N/A	N/A
Inspection Cost (Measured as Percentage of Conversion Cost)	10	10	10	N/A	N/A
Scrap Cost (Measured as Percentage of Conversion Cost)	200	200	200	N/A	N/A
Reworking Cost (Measured as Percentage of Conversion Cost)	25	25	25	N/A	N/A

Optimum Total Cost ($) = 472.33071

Binding Constraint: 31, 32, 33, 34, 36

N/A : No Available Information

Table E-8 Model Verification Results: Effects of Changing in Constraint 51 and 54 on Semi-Tolerance Zones, Costs and Binding Constraints

Condition Changed / Values from Optimal Solution	Value of Constraint					
	Original Condition	P_{TotalL} Changes from 2.5 to 3	P_{TotalL} Changes from 2.5 to 5.	P_{TotalU} Changes from 2.5 to 3	P_{TotalU} Changes from 2.5 to 4	P_{TotalU} Changes from 2.5 to 4, P_{TotalL} Changes from 2.5 to 4
Lower Semi-Tolerance Zone for Part 1 (mm)	0.032	0.032	Infeasible	0.032	0.032	0.032
Upper Semi-Tolerance Zone for Part 1 (mm)	0.032	0.032		0.032	0.032	0.032
Lower Semi-Tolerance Zone for Part 2 (mm)	0.032	0.032		0.032	0.032	0.033
Upper Semi-Tolerance Zone for Part 2 (mm)	0.034	0.032		0.034	0.034	0.045
Lower Semi-Tolerance Zone for Part 3 (mm)	0.032	0.032		0.032	0.032	0.032
Upper Semi-Tolerance Zone for Part 3 (mm)	0.032	0.032		0.032	0.032	0.032
Conversion Cost for Lower Side of Part 1 ($)	21.21695501	21.21695501		21.21695501	21.21695501	21.216955
Conversion Cost for Upper Side of Part 1 ($)	39.32350508	39.32350508		39.32350508	39.32350508	39.3235051
Conversion Cost for Lower Side of Part 2 ($)	16.91829158	16.97356401		16.91829158	16.91829158	16.6363404
Conversion Cost for Upper Side of Part 2 ($)	30.53156717	31.45880407		30.53156717	30.53156717	25.5630409
Conversion Cost for Lower Side of Part 3 ($)	29.88586386	29.88586386		29.88586386	29.88586386	29.8858639
Conversion Cost for Upper Side of Part 3 ($)	16.12488581	16.12488581		16.12488581	16.12488581	16.1248858
Scrap Cost for Part1 ($)	0.011336919	0.011336919		0.011336919	0.011336919	0.01133692
Scrap Cost for Part2 ($)	0.055864265	0.07797913		0.055864265	0.055864265	0.01359747
Scrap Cost for Part3 ($)	0.066953133	0.066953133		0.066953133	0.066953133	0.06695313
Reworking Cost for Part1 ($)	0.008466594	0.008466594		0.008466594	0.008466594	0.00846659
Reworking Cost for Part2 ($)	0	0		0	0	0
Reworking Cost for Part3 ($)	0.001400804	0.001400804		0.001400804	0.001400804	0.0014008
Expected Quality Loss for Lower Side of Part 1 ($)	42.02655697	42.02655697		42.02655697	42.02655697	42.026557
Expected Quality Loss for Upper Side of Part 1 ($)	70.06836953	70.06836953		70.06836953	70.06836953	70.0683695
Expected Quality Loss for Lower Side of Part 2 ($)	31.87420454	31.72706789		31.87420454	31.87420454	33.7593686
Expected Quality Loss for Upper Side of Part 2 ($)	55.28189118	52.89664624		55.28189118	55.28189118	62.8188093
Expected Quality Loss for Lower Side of Part 3 ($)	72.06556918	72.06556918		72.06556918	72.06556918	72.0655692
Expected Quality Loss for Upper Side of Part 3 ($)	61.12207857	60.71758183		61.12207857	61.12207857	63.5784704
Total Cost for Part1 ($)	179.2691863	179.2691863		179.2691863	179.2691863	179.269186
Total Cost for Part2 ($)	140.4682256	139.4589016		140.4682256	140.4682256	143.269447
Total Cost for Part3 ($)	152.5933029	152.5933029		152.5933029	152.5933029	152.593303
Grand Total Cost ($)	472.3307149	471.3213909		472.3307149	472.3307149	475.131936
Binding Constraints	31, 32, 33, 34, 36	31, 32, 33, 34, 35, 36		31, 32, 33, 34, 36	31, 32, 33, 34, 36	31, 32, 33 34, 36

Table E-9 Data for a Product Used as an Original Condition for Sensitivity Analysis

Information	Side of Part 1		Side of Part 2		Side of Part 3		Side of Envelope		Side of Gap	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Allowable Minimum Semi-Tolerance Zone (mm)	0.019	0.019	0.019	0.019	0.019	0.019	N/A	N/A	N/A	N/A
Allowable Maximum Semi-Tolerance Zone (mm)	0.085	0.085	0.085	0.085	0.085	0.085	N/A	N/A	N/A	N/A
Allowable Minimum Semi-Tolerance Zone Based on The Capability of the Machine (mm)	0.019	0.019	0.019	0.019	0.019	0.019	N/A	N/A	N/A	N/A
Specified Minimum C*pm Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone	4	4	4	4	4	4	N/A	N/A	N/A	N/A
Specified Minimum Proportion of Conforming Part (Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone)	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	3	3
Fixed Conversion Cost	280.7	280.7	280.7	280.7	280.7	280.7	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 1st Order	2407	2407	2407	2407	2407	2407	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 2nd Order	282.3	282.3	282.3	282.3	282.3	282.3	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 3rd Order	45960	45960	45960	45960	45960	45960	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 4th Order	106100	106100	106100	106100	106100	106100	N/A	N/A	N/A	N/A
Quality Loss Coefficient, K	20340	13560	15570	10380	12320	18480	N/A	N/A	N/A	N/A
Optimum Semi-Tolerance Zone (mm)	0.078	0.085	0.073	0.085	0.085	0.069	0.075	0.075	0.16	0.16

Table E-9 Data for a Product Used as an Original Condition for Sensitivity Analysis (Continue)

Information	Part 1	Part 2	Part 3	Envelope	Gap
Nominal Size (mm)	50.455	40.725	38.75	130.1	0.17
Process Mean of the Dimension (mm)	50.459	40.729	38.746	130.106	0.172
Minimum allowable Process Standard Deviation of the Dimension (mm)	0.012	0.012	0.012	N/A	N/A
Maximum allowable Process Standard Deviation of the Dimension (mm)	0.0156	0.0156	0.0156	N/A	0.0295
Optimum Process Standard Deviation of the Dimension (mm)	0.0154091	0.0152727	0.0151636	0.013	0.0295
Mean Offsets from the Nominal Size (Numbers of Standard Deviation)	0.25959	0.2619048	0.263789	0.4615385	0.0677966
Inspection Strategy	IIR	IWR	NI	NI	NI
Multiplier for Determining Raw Conversion Cost	25	20	19	N/A	N/A
Inspection Cost (Measured as Percentage of Conversion Cost)	10	10	10	N/A	N/A
Scrap Cost (Measured as Percentage of Conversion Cost)	200	200	200	N/A	N/A
Reworking Cost (Measured as Percentage of Conversion Cost)	25	25	25	N/A	N/A

Optimum Total Cost ($) = 93.965244

Binding Constraint: 11, 23, 24, 25

N/A : No Available Information

Table E-10 Minimum Total Cost for Screening Factors (Quality Loss Coefficients and Constraints) Using Plackett-Burman Design

Run	Minimum	Values of Quality Loss Coefficient or Constraints																												
	Total Cost	KL1	KL2	KL3	KU1	KU2	KU3	11	21	22	23	24	25	26	31	32	33	34	35	36	41	42	43	51	54
1	105.7248	24408	12456	14784	16272	12456	22176	0.03	0.08	0.08	0.08	0.09	0.08	0.08	3.75	4.25	3.75	3.75	4.25	4.25	0.0161	0.0151	0.0161	2.75	3.25
2	108.2411	24408	18684	9856	16272	12456	22176	0.03	0.08	0.08	0.08	0.08	0.09	0.09	3.75	3.75	4.25	3.75	3.75	3.75	0.0161	0.0161	0.0161	3.25	2.75
3	99.94393	16272	18684	14784	16272	12456	22176	0.03	0.08	0.08	0.09	0.08	0.08	0.08	4.25	3.75	3.75	4.25	3.75	4.25	0.0151	0.0161	0.0151	3.25	3.25
4	92.68949	16272	12456	9856	16272	8304	22176	0.03	0.09	0.09	0.08	0.08	0.09	0.08	4.25	3.75	3.75	3.75	4.25	4.25	0.0151	0.0161	0.0161	3.25	2.75
5	91.90341	16272	12456	9856	16272	12456	14784	0.03	0.09	0.09	0.09	0.08	0.08	0.08	3.75	4.25	4.25	3.75	3.75	4.25	0.0161	0.0151	0.0151	3.25	3.25
6	94.10065	16272	12456	9856	10848	12456	22176	0.03	0.09	0.09	0.08	0.09	0.08	0.09	3.75	3.75	3.75	4.25	3.75	3.75	0.0161	0.0161	0.0161	2.75	3.25
7	93.89571	24408	18684	14784	10848	8304	14784	0.03	0.08	0.09	0.08	0.08	0.09	0.08	3.75	4.25	3.75	4.25	3.75	4.25	0.0151	0.0161	0.0161	2.75	3.25
8	93.42147	24408	18684	14784	10848	8304	14784	0.03	0.09	0.08	0.09	0.08	0.08	0.09	3.75	3.75	3.75	3.75	4.25	4.25	0.0161	0.0151	0.0161	3.25	2.75
9	100.1516	24408	18684	14784	10848	8304	14784	0.03	0.09	0.09	0.08	0.09	0.08	0.08	4.25	3.75	4.25	3.75	3.75	3.75	0.0161	0.0161	0.0151	3.25	3.25
10	91.78635	24408	18684	9856	16272	8304	22176	0.03	0.08	0.09	0.09	0.08	0.09	0.09	4.25	4.25	3.75	3.75	3.75	3.75	0.0161	0.0151	0.0151	2.75	3.25
11	92.81286	16272	18684	14784	16272	12456	14784	0.03	0.09	0.08	0.09	0.09	0.08	0.09	4.25	4.25	3.75	3.75	3.75	3.75	0.0151	0.0161	0.0161	2.75	2.75
12	92.81596	24408	12456	14784	10848	12456	22176	0.03	0.09	0.09	0.08	0.09	0.09	0.09	4.25	4.25	3.75	3.75	3.75	4.25	0.0151	0.0151	0.0151	3.25	2.75
13	94.60883	24408	12456	14784	16272	12456	14784	0.03	0.08	0.09	0.08	0.08	0.08	0.09	3.75	4.25	4.25	4.25	4.25	3.75	0.0151	0.0161	0.0151	3.25	2.75
14	93.32586	24408	18684	9856	10848	12456	22176	0.03	0.09	0.08	0.08	0.08	0.08	0.09	4.25	3.75	4.25	4.25	4.25	4.25	0.0151	0.0151	0.0151	2.75	3.25
15	102.9652	16272	18684	14784	16272	8304	22176	0.03	0.09	0.09	0.08	0.08	0.08	0.08	4.25	4.25	4.25	4.25	4.25	3.75	0.0161	0.0151	0.0161	2.75	2.75
16	90.57334	24408	12456	14784	16272	8304	22176	0.03	0.09	0.08	0.09	0.09	0.09	0.08	3.75	3.75	4.25	3.75	4.25	3.75	0.0151	0.0161	0.0151	2.75	3.25
17	94.06456	24408	18684	9856	16272	12456	14784	0.03	0.09	0.09	0.09	0.09	0.09	0.08	3.75	3.75	4.25	4.25	3.75	4.25	0.0151	0.0151	0.0161	2.75	2.75
18	90.45764	16272	18684	14784	10848	12456	22176	0.03	0.08	0.09	0.09	0.09	0.09	0.08	3.75	3.75	3.75	4.25	4.25	3.75	0.0161	0.0151	0.0151	3.25	2.75
19	98.34199	16272	18684	9856	10848	8304	22176	0.03	0.08	0.09	0.09	0.09	0.08	0.09	3.75	4.25	4.25	3.75	4.25	4.25	0.0151	0.0161	0.0161	3.25	3.25
20	92.19655	16272	12456	14784	16272	8304	14784	0.03	0.08	0.08	0.08	0.09	0.09	0.09	4.25	3.75	4.25	4.25	3.75	4.25	0.0161	0.0151	0.0161	3.25	3.25
21	105.7847	24408	12456	9856	10848	12456	14784	0.03	0.09	0.08	0.09	0.08	0.09	0.08	4.25	4.25	3.75	4.25	4.25	3.75	0.0161	0.0161	0.0161	3.25	3.25
22	94.3662	16272	12456	14784	10848	12456	14784	0.03	0.08	0.09	0.09	0.08	0.09	0.09	4.25	3.75	4.25	3.75	4.25	3.75	0.0151	0.0151	0.0161	2.75	3.25
23	91.02232	24408	12456	9856	10848	8304	22176	0.03	0.08	0.08	0.09	0.09	0.08	0.08	4.25	4.25	4.25	4.25	3.75	3.75	0.0151	0.0151	0.0161	3.25	2.75
24	90.25782	16272	18684	9856	16272	8304	14784	0.03	0.09	0.08	0.08	0.09	0.09	0.09	3.75	4.25	3.75	4.25	4.25	3.75	0.0151	0.0151	0.0151	3.25	3.25
25	99.23519	16272	12456	14784	10848	8304	22176	0.03	0.09	0.08	0.09	0.08	0.09	0.09	3.75	4.25	4.25	4.25	3.75	4.25	0.0161	0.0161	0.0151	2.75	2.75
26	99.37737	24408	12456	9856	16272	8304	14784	0.03	0.08	0.09	0.09	0.09	0.08	0.09	4.25	3.75	3.75	4.25	4.25	4.25	0.0161	0.0161	0.0151	2.75	2.75
27	92.56969	16272	18684	9856	10848	12456	14784	0.03	0.08	0.08	0.08	0.09	0.09	0.08	4.25	4.25	4.25	3.75	4.25	4.25	0.0161	0.0161	0.0151	2.75	2.75
28	93.46775	16272	12456	9856	10848	8304	14784	0.03	0.08	0.08	0.08	0.08	0.08	0.08	3.75	3.75	3.75	3.75	3.75	3.75	0.0151	0.0151	0.0151	2.75	2.75

Table E-11 Estimated Effects and Coefficients for Total Cost

Term	Effect	Coef	SE Coef	T	P
Constant		95.718	0.3452	277.27	0.000
KL1	2.106	1.053	0.3452	3.05	0.055
KL2	0.312	0.156	0.3452	0.45	0.682
KL3	0.445	0.223	0.3452	0.65	0.565
KU1	1.013	0.507	0.3452	1.47	0.238
KU2	1.524	0.762	0.3452	2.21	0.114
KU3	1.596	0.798	0.3452	2.31	0.104
11	-6.384	-3.192	0.3452	-9.25	0.003
21	-0.850	-0.425	0.3452	-1.23	0.306
22	-1.218	-0.609	0.3452	-1.76	0.176
23	-0.994	-0.497	0.3452	-1.44	0.245
24	-2.226	-1.113	0.3452	-3.22	0.048
25	-1.588	-0.794	0.3452	-2.30	0.105
26	-0.738	-0.369	0.3452	-1.07	0.364
31	0.251	0.125	0.3452	0.36	0.740
32	0.525	0.262	0.3452	0.76	0.502
33	0.502	0.251	0.3452	0.73	0.520
34	0.169	0.085	0.3452	0.25	0.822
35	0.630	0.315	0.3452	0.91	0.428
36	-0.078	-0.039	0.3452	-0.11	0.917
41	3.981	1.990	0.3452	5.77	0.010
42	3.182	1.591	0.3452	4.61	0.019
43	2.797	1.398	0.3452	4.05	0.027
51	0.255	0.128	0.3452	0.37	0.736
54	0.329	0.164	0.3452	0.48	0.666

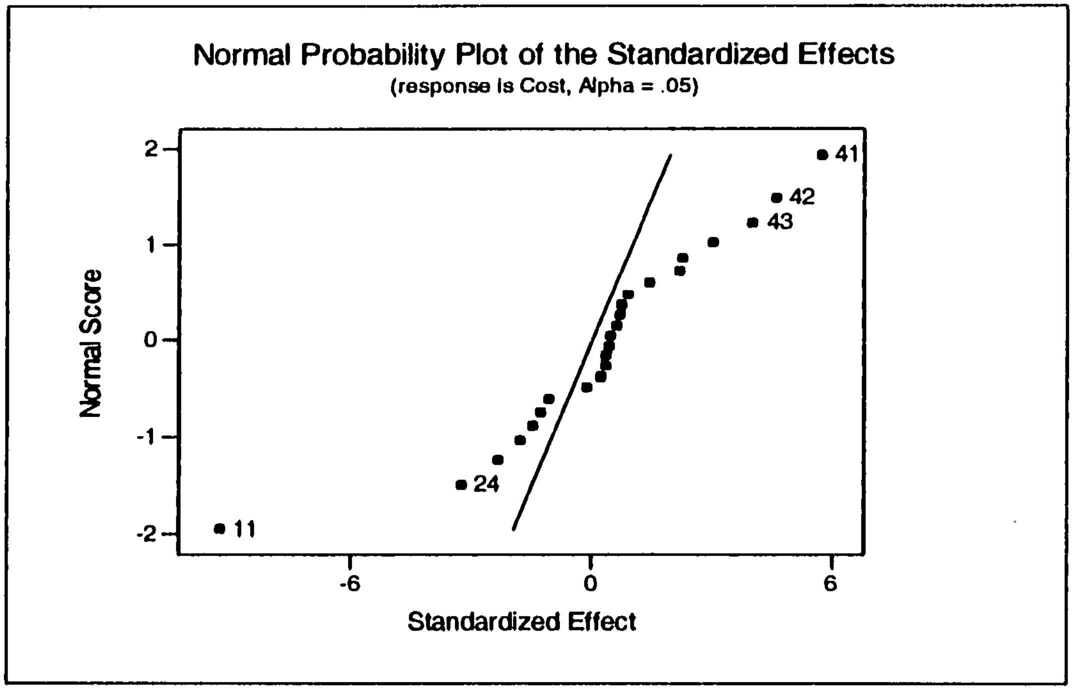


Figure E-1 Normal Probability Plot of Standardized Effects for Screening Singinificant Factors

Table E-12 Minimum Total Cost for Determining Two-Factor Interaction Effects

Run	Minimum Total Cost	Values of Constraints				
		11	24	41	42	43
1	96.42619	0.029	0.08	0.0151	0.0151	0.0161
2	91.86219	0.03	0.08	0.0151	0.0151	0.0151
3	91.22481	0.029	0.09	0.0151	0.0151	0.0151
4	90.6409	0.03	0.09	0.0151	0.0151	0.0161
5	96.85729	0.029	0.08	0.0161	0.0151	0.0151
6	93.72939	0.03	0.08	0.0161	0.0151	0.0161
7	99.77069	0.029	0.09	0.0161	0.0151	0.0161
8	90.6915	0.03	0.09	0.0161	0.0151	0.0151
9	96.59085	0.029	0.08	0.0151	0.0161	0.0151
10	93.54921	0.03	0.08	0.0151	0.0161	0.0161
11	99.5279	0.029	0.09	0.0151	0.0161	0.0161
12	90.56704	0.03	0.09	0.0151	0.0161	0.0151
13	107.1334	0.029	0.08	0.0161	0.0161	0.0161
14	93.7094	0.03	0.08	0.0161	0.0161	0.0151
15	99.9776	0.029	0.09	0.0161	0.0161	0.0151
16	95.17522	0.03	0.09	0.0161	0.0161	0.0161

Table E-13 Estimated Effects and Coefficients for Total Cost (coded units)

Term	Effect	Coef
Constant		95.465
11	-5.948	-2.974
24	-1.535	-0.768
41	3.332	1.666
42	3.128	1.564
43	3.059	1.530
11*24	0.091	0.046
11*41	-1.660	-0.830
11*42	-1.609	-0.805
11*43	-1.493	-0.746
24*41	0.082	0.041
24*42	0.102	0.051
24*43	0.104	0.052
41*42	0.608	0.304
41*43	0.584	0.292
42*43	0.576	0.288

Table E-14 Estimated Effects and Coefficients for Total Cost (Coded Units)

Term	Effect	Coef	SE Coef	T	P
Constant		95.465	0.04758	2006.20	0.000
11	-5.948	-2.974	0.04758	-62.50	0.000
24	-1.535	-0.768	0.04758	-16.13	0.000
41	3.332	1.666	0.04758	35.01	0.000
42	3.128	1.564	0.04758	32.87	0.000
43	3.059	1.530	0.04758	32.14	0.000
11*41	-1.660	-0.830	0.04758	-17.45	0.000
11*42	-1.609	-0.805	0.04758	-16.91	0.000
11*43	-1.493	-0.746	0.04758	-15.69	0.000
41*42	0.608	0.304	0.04758	6.39	0.003
41*43	0.584	0.292	0.04758	6.14	0.004
42*43	0.576	0.288	0.04758	6.05	0.004

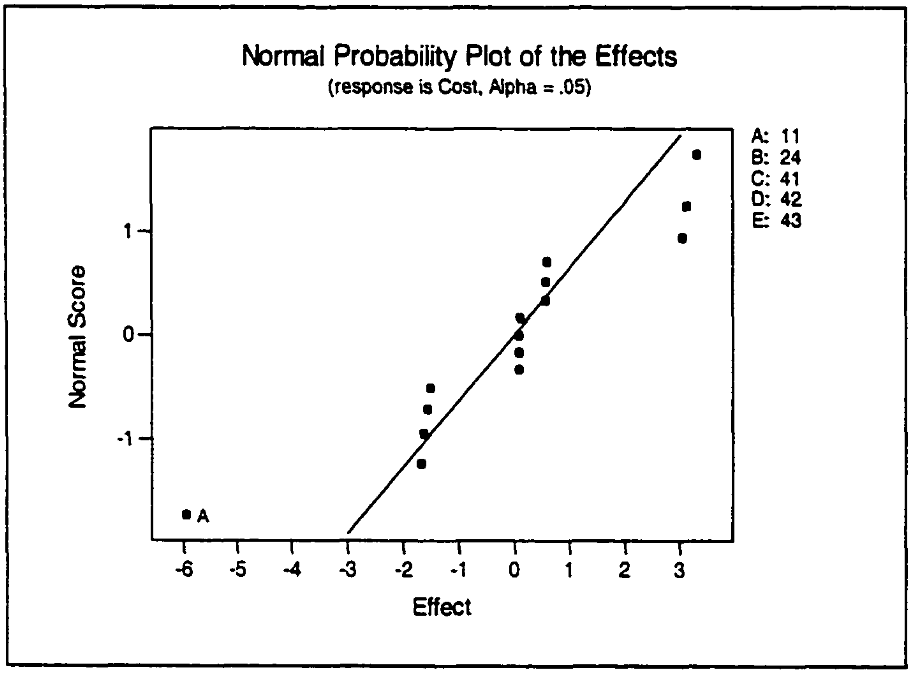


Figure E-2 Normal Probability Plot of Two-Factor Interaction Effects for Preliminary Study

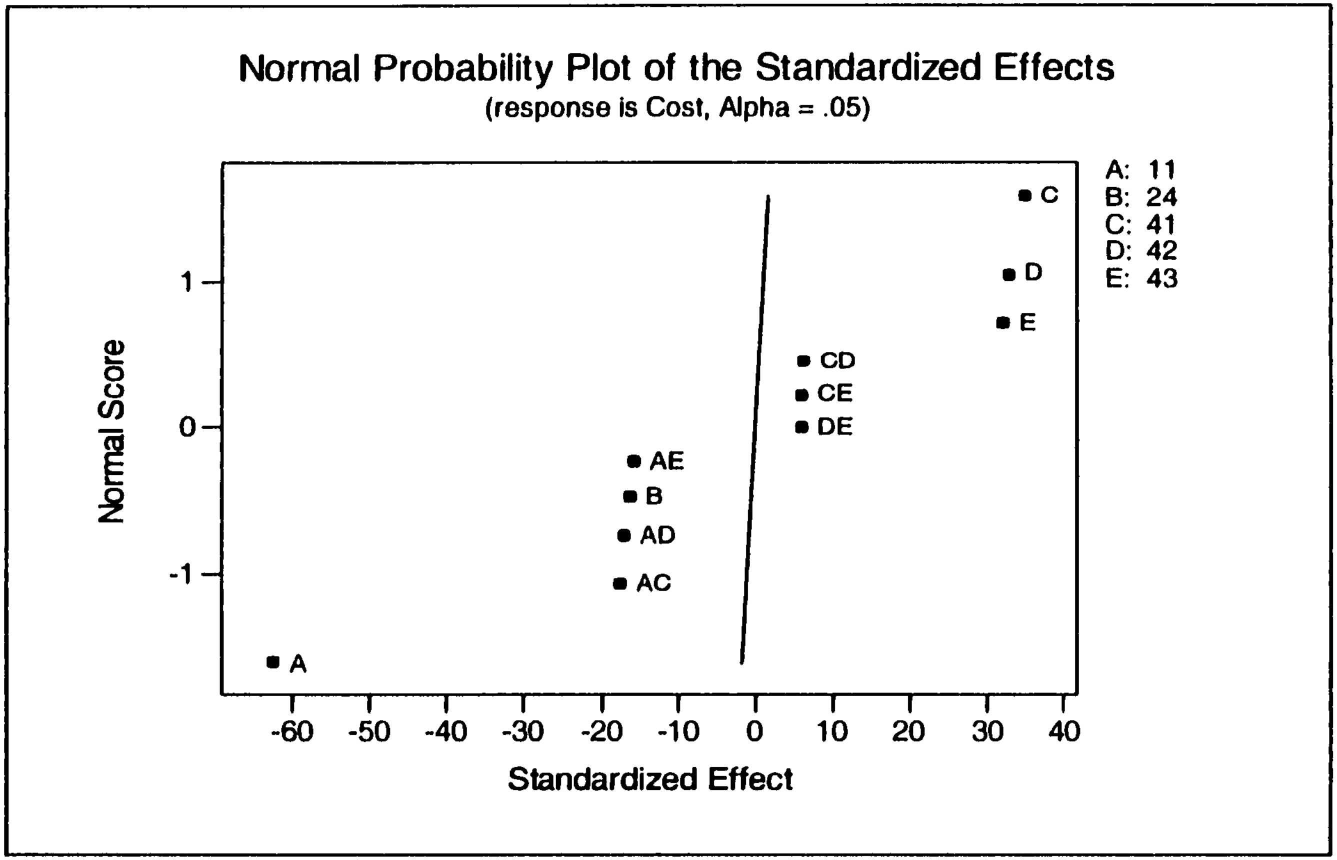


Figure E-3 Normal Probability Plot of the Effects for Two-Factor Interaction Effect

Table E-15 Data for Determining Optimal Solutions for Various Numbers of Standard Deviations in Semi-tolerance Zones (For a Case with L_{Lj} and L_{Uj} equal 4)

Information	Side of Part 1		Side of Part 2		Side of Part 3		Side of Envelope		Side of Gap	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Allowable Minimum Semi-Tolerance Zone (mm)	0.04	0.04	0.04	0.04	0.04	0.04	N/A	N/A	N/A	N/A
Allowable Maximum Semi-Tolerance Zone (mm)	0.085	0.085	0.085	0.085	0.085	0.085	N/A	N/A	N/A	N/A
Allowable Minimum Semi-Tolerance Zone Based on The Capability of the Machine (mm)	0.019	0.019	0.019	0.019	0.019	0.019	N/A	N/A	N/A	N/A
Specified Minimum C*pm Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone	4	4	4	4	4	4	N/A	N/A	N/A	N/A
Specified Minimum Proportion of Conforming Part (Measured as the Specified Minimum Number of Standard Deviations in Semi-Tolerance Zone)	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	3	3
Fixed Conversion Cost	280.7	280.7	280.7	280.7	280.7	280.7	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 1st Order	2407	2407	2407	2407	2407	2407	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 2nd Order	282.3	282.3	282.3	282.3	282.3	282.3	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 3rd Order	45960	45960	45960	45960	45960	45960	N/A	N/A	N/A	N/A
Conversion Cost Coefficient for the 4th Order	106100	106100	106100	106100	106100	106100	N/A	N/A	N/A	N/A
Quality Loss Coefficient, K	305100	203400	233550	155700	184800	277200	N/A	N/A	N/A	N/A
Optimum Semi-Tolerance Zone (mm)	0.059	0.075	0.059	0.076	0.063	0.057	0.075	0.075	0.16	0.16

Table E-15 Data for Determining Optimal Solutions for Various Numbers of Standard Deviations in Semi-tolerance Zones (For a Case with L_{Lj} and L_{Uj} equal 4)

Information	Part 1	Part 2	Part 3	Envelope	Gap
Nominal Size (mm)	50.455	40.725	38.75	130.1	0.17
Process Mean of the Dimension (mm)	50.459	40.729	38.746	130.106	0.172
Minimum allowable Process Standard Deviation of the Dimension (mm)	0.012	0.012	0.012	N/A	N/A
Maximum allowable Process Standard Deviation of the Dimension (mm)	0.0156	0.0156	0.0156	N/A	0.03
Optimum Process Standard Deviation of the Dimension (mm)	0.014618	0.014645	0.014236	0.013	0.02884
Mean Offsets from the Nominal Size (Numbers of Standard Deviation)	0.273632	0.273122	0.280971	0.461538	0.069348
Inspection Strategy	IIR	IWR	NI	NI	NI
Multiplier for Determining Raw Conversion Cost	25	20	19	N/A	N/A
Inspection Cost (Measured as Percentage of Conversion Cost)	10	10	10	N/A	N/A
Scrap Cost (Measured as Percentage of Conversion Cost)	200	200	200	N/A	N/A
Reworking Cost (Measured as Percentage of Conversion Cost)	25	25	25	N/A	N/A

Optimum Total Cost ($) =285.6398

Binding Constraint: 31, 32, 36

N/A : No Available Information

Table E-16 Optimal Solutions for Various Numbers of Standard Deviations in Semi-tolerance Zones

Values from Optimal Solution	Number of Standard Deviations in every Semi-tolerance Zone								
	3	3.25	3.5	3.75	4	4.25	4.5	4.75	5
Lower Semi-Tolerance Zone for Part 1 (mm)	0.05	0.05	0.051	0.055	0.059	0.063	0.067	0.071	0.076
Upper Semi-Tolerance Zone for Part 1 (mm)	0.075	0.075	0.075	0.075	0.075	0.074	0.074	0.074	0.076
Lower Semi-Tolerance Zone for Part 2 (mm)	0.052	0.052	0.052	0.055	0.059	0.063	0.067	0.071	0.076
Upper Semi-Tolerance Zone for Part 2 (mm)	0.076	0.076	0.076	0.076	0.076	0.075	0.075	0.075	0.076
Lower Semi-Tolerance Zone for Part 3 (mm)	0.068	0.068	0.067	0.067	0.063	0.063	0.066	0.071	0.076
Upper Semi-Tolerance Zone for Part 3 (mm)	0.042	0.046	0.05	0.054	0.057	0.061	0.066	0.071	0.076
Conversion Cost for Lower Side of Part 1 ($)	16.345197	16.345197	16.1087112	15.2195655	14.4218815	13.7082892	13.0907286	12.5588244	12.0190019
Conversion Cost for Upper Side of Part 1 ($)	20.2798529	20.2798529	20.272736	20.2451873	20.2185762	20.4345617	20.4085835	20.3829951	19.8786402
Conversion Cost for Lower Side of Part 2 ($)	12.708799	12.708799	12.708799	12.1818165	11.5432591	10.9720363	10.477662	10.0518544	9.61520149
Conversion Cost for Upper Side of Part 2 ($)	16.0250871	16.0250871	16.0250871	16.0088852	15.9879028	16.1540376	16.1335668	16.1134035	15.9029121
Conversion Cost for Lower Side of Part 3 ($)	16.9047242	16.8761386	17.0845178	17.0605919	18.0467113	18.0223455	17.231816	16.0814519	15.1077665
Conversion Cost for Upper Side of Part 3 ($)	13.9570468	13.1348674	12.3703762	11.6808611	11.1857094	10.6186998	10.0203224	9.53095169	9.13444142
Scrap Cost for Part1 ($)	0.00025189	0.00025189	0.00019471	6.7761E-05	2.2668E-05	7.0659E-06	2.1897E-06	6.561E-07	1.5231E-07
Scrap Cost for Part2 ($)	0.00015465	0.00015465	0.00015465	7.0597E-05	2.4567E-05	9.2445E-06	4.5438E-06	3.3398E-06	2.5608E-06
Scrap Cost for Part3 ($)	0	0	0	0	0	0	0	0	0
Reworking Cost for Part1 ($)	1.4334E-07	1.4334E-07	1.4938E-07	1.7592E-07	2.0674E-07	3.2779E-07	3.8244E-07	4.4556E-07	3.0106E-07
Reworking Cost for Part2 ($)	0	0	0	0	0	0	0	0	0
Reworking Cost for Part3 ($)	0	0	0	0	0	0	0	0	0
Expected Quality Loss for Lower Side of Part 1 ($)	26.2736574	26.2736574	26.4172171	26.9654177	27.4916053	27.8827268	28.4005118	28.9212589	29.8426595
Expected Quality Loss for Upper Side of Part 1 ($)	42.7844929	42.7844929	42.9133676	43.432277	43.9555634	44.3500126	44.8794579	45.4121552	46.3533185
Expected Quality Loss for Lower Side of Part 2 ($)	20.4238855	20.4238855	20.4238855	20.7373962	21.1415764	21.4418735	21.8392345	22.2388097	22.8441381
Expected Quality Loss for Upper Side of Part 2 ($)	33.0483265	33.0483265	33.0483265	33.3470606	33.7482569	34.0507427	34.4566851	34.8651241	35.4828165
Expected Quality Loss for Lower Side of Part 3 ($)	37.1236208	37.5850679	37.9330892	38.3996983	38.2827695	38.7515905	39.6980789	40.8977725	42.1158868
Expected Quality Loss for Upper Side of Part 3 ($)	33.0469547	33.0469547	33.0469547	33.3466961	33.7485247	34.0515262	34.4577011	34.8663593	35.484002
Total Cost for Part1 ($)	109.352007	109.352007	109.35505	109.410623	109.552246	109.790064	110.129281	110.569448	111.283399
Total Cost for Part2 ($)	85.0825797	85.0825797	85.0825797	85.0956406	85.1746027	85.3314824	85.5683621	85.8857844	86.3969309
Total Cost for Part3 ($)	90.2793293	90.3282669	90.4517371	90.6508126	90.9129647	91.2395977	91.7086757	92.4326141	93.4718631
Grand Total Cost ($)	284.713916	284.762854	284.889366	285.157076	285.639814	286.361144	287.406319	288.887846	291.152193
Binding Constraint(s)	36	36	31, 36	31, 32, 36	31, 32, 36	31, 32, 36	31, 32, 33 36	31, 32, 33, 36	31, 32, 33 34, 35, 36

Appendix F

Concept for The Relationship Between The Process Standard Deviation And The Tolerance for Verification

Changing in the Allowable Maximum Tolerance

In order to clearly study the effect of change in one constraint at a time, the values of other constraints must remain the same. The model of this research assumes that the process standard deviation has an increasing linear relationship with the tolerance. As a result, the allowable maximum process standard deviation remains the same when the allowable maximum tolerance is decreased or increased as shown in Figure F-1.

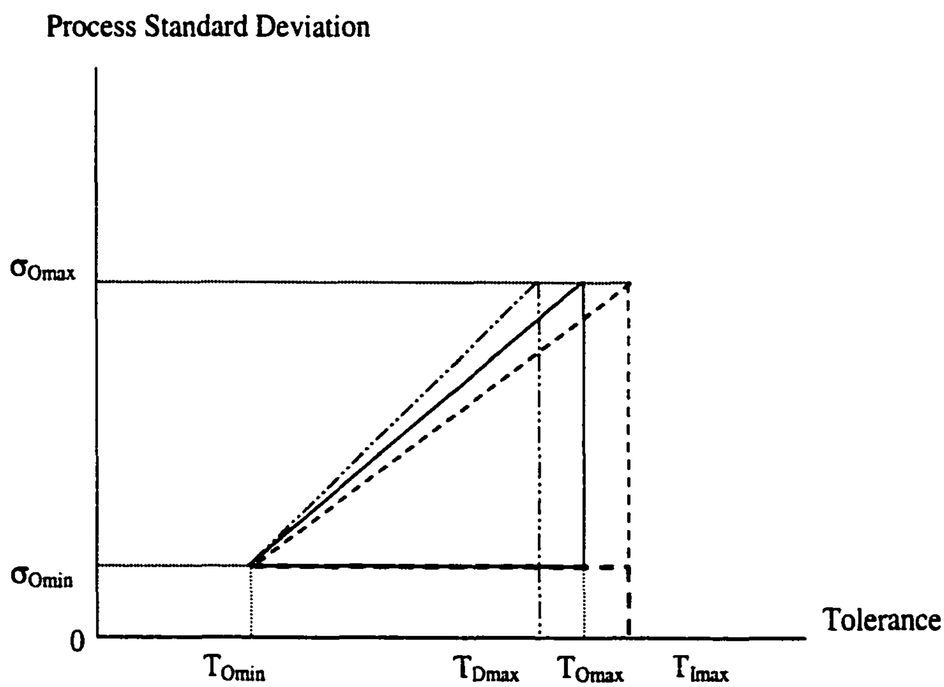


Figure F-1. Concept for Changes in The Allowable Maximum Tolerance

Letting

σ_{Omin} = the allowable minimum process standard deviation of each part that depends on the capability of the machine and/or process used to manufacture the part for the chosen original condition

σ_{Omax} = the allowable maximum process standard deviation of each part depending on the economic condition that depends on the capability of the machine and/or the process used to manufacture the part for the chosen original condition

$T_{Omin} = \Delta_{OMinA_L} + \Delta_{OMinA_U}$

= the allowable minimum value of the tolerance for each part based on the capability of the machine and/or process used to manufactured the part for the chosen original condition

$T_{Omax} = \Delta_{OMax_L} + \Delta_{OMax_U}$

= the allowable maximum value of the tolerance for each part based on the capability of the machine and/or process used to manufactured the part for the chosen original condition

$T_{Dmax} = \Delta_{DMax_L} + \Delta_{OMax_U}$, or

$= \Delta_{OMax_L} + \Delta_{DMax_U}$

= the allowable maximum value of the tolerance for each part for a new condition with a decreasing in the allowable maximum value of the lower or the upper semi-tolerance zone, respectively

$T_{Imax} = \Delta_{IMax_L} + \Delta_{OMax_U}$

$= \Delta_{OMax_L} + \Delta_{IMax_U}$

= the allowable maximum value of the tolerance for each part for a new condition with an increasing in the allowable maximum value of the lower or the upper semi-tolerance zone, repectively

where

Δ_{OMinA_L} = the allowable minimum value of the lower semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured that part for the chosen original condition

Δ_{OMinA_U} = the allowable minimum value of the upper semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured that part for the chosen original condition

Δ_{OMax_L} = the allowable maximum value of the lower semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured the part for the chosen original condition

Δ_{DMax_L} = the allowable maximum value of the lower semi-tolerance zone for each part for a new condition with a decreasing in the maximum value of the lower semi-tolerance zone

Δ_{IMax_L} = the allowable maximum value of the lower semi-tolerance zone for each part for a new condition with an increasing in the maximum value of the lower semi-tolerance zone

Δ_{OMax_U} = the allowable maximum value of the upper semi-tolerance zone for each part based on the capability of the machine and/or process used to manufactured the part for the chosen original condition

Δ_{DMax_U} = the allowable maximum value of the upper semi-tolerance zone for each part for a new condition with a decreasing in the maximum value of the upper semi-tolerance zone

Δ_{IMax_U} = the allowable maximum value of the upper semi-tolerance zone for each part for a new condition with an increasing in the maximum value of the upper semi-tolerance zone

Finally, the process standard deviation that has an increasing linear relationship with the tolerance for the chosen original condition can be expressed as

$$\sigma = \sigma_{O\min} + \left[\left\{ \frac{\sigma_{O\max} - \sigma_{O\min}}{\left(\Delta_{O\max L} + \Delta_{O\max U}\right) - \left(\Delta_{OMinA_L} + \Delta_{OMinA_U}\right)} \right\} * \left\{ \left(\Delta_L + \Delta_U\right) - \left(\Delta_{OMinA_L} + \Delta_{OMinA_U}\right) \right\} \right]$$

where

σ = the optimum standard deviation for each part. It affects the expected quality loss of the objective function in the model OTA

Δ_L = the lower semi-tolerance zone being optimized

Δ_U = the upper semi-tolerance zone being optimized.

The process standard deviation for a new condition with a decreasing in the allowable maximum lower semi-tolerance zone can be expressed as

$$\sigma = \sigma_{\mathrm{Omin}} + \left[\begin{array}{l} \left\{ \dfrac{\sigma_{\mathrm{Omax}} - \sigma_{\mathrm{Omin}}}{\left(\Delta_{\mathrm{Dmax}L} + \Delta_{\mathrm{Omax}U}\right) - \left(\Delta_{\mathrm{O}MinA_L} + \Delta_{\mathrm{O}MinA_U}\right)} \right\} \\ * \left\{ \left(\Delta_L + \Delta_U\right) - \left(\Delta_{\mathrm{O}MinA_L} + \Delta_{\mathrm{O}MinA_U}\right) \right\} \end{array} \right].$$

The process standard deviation for a new condition with a decreasing in the allowable maximum upper semi-tolerance zone can be expressed as

$$\sigma = \sigma_{\mathrm{Omin}} + \left[\begin{array}{l} \left\{ \dfrac{\sigma_{\mathrm{Omax}} - \sigma_{\mathrm{Omin}}}{\left(\Delta_{\mathrm{Omax}L} + \Delta_{\mathrm{Dmax}U}\right) - \left(\Delta_{\mathrm{O}MinA_L} + \Delta_{\mathrm{O}MinA_U}\right)} \right\} \\ * \left\{ \left(\Delta_L + \Delta_U\right) - \left(\Delta_{\mathrm{O}MinA_L} + \Delta_{\mathrm{O}MinA_U}\right) \right\} \end{array} \right].$$

The process standard deviation for a new condition with an increasing in the allowable maximum lower semi-tolerance zone can be expressed as

$$\sigma = \sigma_{\mathrm{Omin}} + \left[\begin{array}{l} \left\{ \dfrac{\sigma_{\mathrm{Omax}} - \sigma_{\mathrm{Omin}}}{\left(\Delta_{\mathrm{Imax}L} + \Delta_{\mathrm{Omax}U}\right) - \left(\Delta_{\mathrm{O}MinA_L} + \Delta_{\mathrm{O}MinA_U}\right)} \right\} \\ * \left\{ \left(\Delta_L + \Delta_U\right) - \left(\Delta_{\mathrm{O}MinA_L} + \Delta_{\mathrm{O}MinA_U}\right) \right\} \end{array} \right].$$

The process standard deviation for a new condition with an increasing in the allowable maximum upper semi-tolerance zone can be expressed as

$$\sigma = \sigma_{O\min} + \left[\left\{ \frac{\sigma_{O\max} - \sigma_{O\min}}{\left(\Delta_{O\max L} + \Delta_{I\max U}\right) - \left(\Delta_{OMinA_L} + \Delta_{OMinA_U}\right)} \right\} * \left\{ \left(\Delta_L + \Delta_U\right) - \left(\Delta_{OMinA_L} + \Delta_{OMinA_U}\right) \right\} \right].$$

The increasing rate (the slope) of the process standard deviation for the case with decreasing the allowable maximum tolerance is greater than that for the original condition. In contrast, the increasing rate of the process standard deviation for the case with increasing the allowable maximum tolerance is smaller than that for the original condition.

Changing in the Allowable Maximum Standard Deviation

For the concept for changing in the allowable maximum process standard deviation, the value of the allowable maximum tolerance must remain the same. At any values of the tolerances, the process standard deviation for the case with decreasing the allowable maximum process standard deviation are the smallest; those for the original condition are, and those for the case with increasing the allowable maximum process standard deviation are the greatest as shown in Figure F-2.

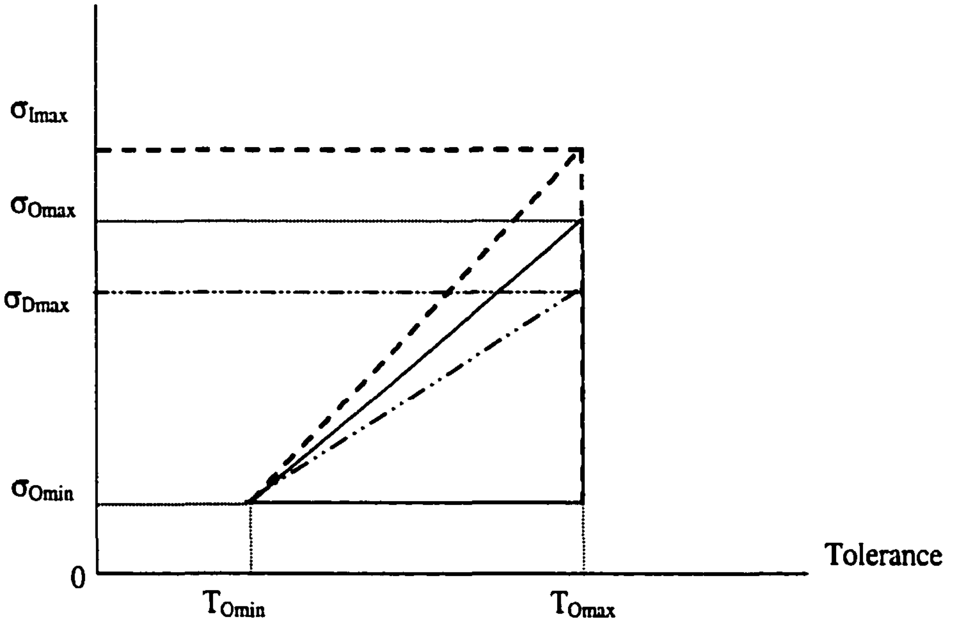


Figure F-2. Concept for Changing The Maximum Process Standard Deviation

Letting

$\sigma_{D\max}$ = the allowable maximum process standard deviation of each part for a new condition with a decreasing in the allowable maximum standard deviation

$\sigma_{I\max}$ = the allowable maximum process standard deviation of each part for a new condition with an increasing in the allowable maximum standard deviation

The process standard deviation for a new condition with a decreasing in the allowable maximum standard deviation can be expressed as

$$\sigma = \sigma_{O\min} + \left[\begin{array}{l} \left\{ \dfrac{\sigma_{D\max} - \sigma_{O\min}}{\left(\Delta_{O\max L} + \Delta_{O\max U}\right) - \left(\Delta_{OMinA_L} + \Delta_{OMinA_U}\right)} \right\} \\ * \left\{ \left(\Delta_L + \Delta_U\right) - \left(\Delta_{OMinA_L} + \Delta_{OMinA_U}\right) \right\} \end{array} \right].$$

The process standard deviation for a new condition with an increasing in the allowable maximum standard deviation can be expressed as

$$\sigma = \sigma_{\mathrm{Omin}} + \left[\begin{array}{l} \left\{ \dfrac{\sigma_{\mathrm{Imax}} - \sigma_{\mathrm{Omin}}}{\left(\Delta_{\mathrm{Omax}L} + \Delta_{\mathrm{Omax}U}\right) - \left(\Delta_{OMinA_L} + \Delta_{OMinA_U}\right)} \right\} \\ * \left\{ \left(\Delta_L + \Delta_U\right) - \left(\Delta_{OMinA_L} + \Delta_{OMinA_U}\right) \right\} \end{array} \right].$$

Appendix G

Appropriate Conditions For Running Evolver For Numerical Examples in This Research

The appropriate conditions for running Evolver for numerical examples in this research are listed as follows:

(1) The semi-tolerance zones in the adjustable cells are set as integer, but they are divided by 1000 in order to transform them to the desired values with tree digits in mm at the final stage.

(2) "Recipe" is chosen as the solving method

(3) Cross over rate is set at 0.5 while the mutation rate is set to be auto changed to appropriate rates

(4) All available optimization operators for searching the best solution are selected because the conditions of the numerical examples in this research are changed all the times. All of the available optimization operators are default parent selection, default mutation, default crossover, default backtrack, arithmetic crossover, heuristic crossover, Cauchy mutation, boundary mutation, non-uniform mutation, linear and local search.

(5) Population size is set at 50, and

Running Evolver is chosen to stop when the changes in the total cost in last 10000 trials are less than 1E-15.

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